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STRUCTURALLY INTELLIGENT SWARMS

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SUMMARY: *The potential afforded by the open search spaces of both agent-based models and evolutionary engines have given architecture yet another set of computational tools to play with, yet more often than not, they are used in isolation from one another. This research explores the set of techniques and results of having combined swarm formations, FEM software and an evolutionary engine within a parametric modeling environment such that they induce structurally intelligent swarm (SIS) morphologies. These morphologies are situated within normative architectural assemblies by means of parametric grafting techniques. Savage gothic materiality, as described by John Ruskin, as well as the work of Eva Hesse are referenced as the basis for these explorations. Speculations are made as to refining the engineering capabilities, expanding on programmatic applications and testing integrated SIS assemblies at larger scales when tackling initial problems of adaptive reuse.*

KEYWORDS: *Swarms*

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1. FROM SWARMING-IN TO SWARMING-AS ARCHITECTURE

The adoption of swarms into architectural design has ushered in a profound capacity to instrumentalize large populations of agents toward discovering and developing intelligent solutions within complex environments. Simple interactions between many thousands (even millions) of autonomous agents enable emergent behaviors to explicitly form and inform a design. While swarms in architecture may appear to be a relatively new phenomenon given the correlation to their prevalence in the discipline of computer science, swarms can be traced, in a sense, to Gothic architectural performance. When describing the Nature of Gothic, John Ruskin correlates savageness with the animal populations of the north. In doing so, Ruskin positions the bodies of animals as well as the material bodies of architecture as inflecting the force of their environment as registered through their textured surfaces:

> ***"And, having once traversed in thought this gradation of the zoned iris of the earth in all its material vastness, let us go down nearer to it, and watch the change in the belt of animal life; the multitude of swift and brilliant creatures that glance in the air and sea, or tread the sands of the southern zone… Let us contrast their delicacy and brilliancy of colour, and swiftness of motion, with the frost-cramped strength, and shaggy covering, and dusky plumage of the northern tribes… and then submissively acknowledge the great laws by which the earth and all that it bears are ruled through their being." (Ruskin, 1853)***

The ability of an animal herd to produce an intricate reading of the land positions swarms as a sensitive and robust population resistant to the "unnatural" stability of visual symmetry all the while adhering to a greater law that governs their tendency to seek out their own conditional balance. The constant production of disequilibrium through the seeking of equilibrium is something that swarms have been designed to perpetually achieve in the architectural project.

Often, swarms are observed and directly translated into human societies with the claim of offering a viable or more desirable model of societal behavior continuing William Morton Wheeler's hierarchical projection of super-superorganisms. In many ways, this could be seen also as a shift away from anthropocentric models of architecture toward the projection of other forms of biological life as a way of organizing the city. This mode of operation is obviously problematic in that it assumes that "natural" systems offer an inherent viability when searching for solutions and are appropriate for direct transference across scales and media. This is certainly understandable given technology's tendency toward remixing and remediation – and there certainly is value in such explorations.

In contrast to techniques of direct transference, there is another set of contemporary agent-based projects that has started to emerge where swarms are not so much imaged as architecture but rather as inflected patterns within and through the architectural body. Projects such as Biothing's Kaohsiung Pop Center or Kokkugia's Babiy Yar Memorial serve as such an example. It could be argued that this is a continuation of Ruskin's implied swarms that offers a more sophisticated and integrated relationship between swarming and the production of architectural form. In particular, the remixing and layering of differential systems, such as in Andrasek's project descriptions (e.g. magnetic stitching + Brownian erosion = resilient adaptive planning logics), in turn produce a kind of synthetic life that at least obscures if not invalidates the premise that 'natural' life is the sole proprietor of complex and viable solution spaces. In the cases of these synthetic swarms, the agents in question do not simply operate as autonomous formations imaged as isolated objects within some site, but rather as integrated effects born out of autonomous behaviors that in turn augment and provide new legibility to the sites they inhabit. This is a radical alternative to the aforementioned super-superorganism model in that a project's success is not so much measured in its relative similarity to any referenced system, but rather in its ability to augment, mutate, transform or otherwise change presently existing bodies of architecture whether historical, conceptual, informatics, material, and so on. Toward this end, this research aims to develop generic grafting techniques such that any swarm can become an integrated part of an otherwise normative architectural assembly.

Adding to this new understanding are new tools that exploit software interoperability, in particular, the Geometry Gym tools developed by Jon Mirtschin. These tools make possible new forms of generative design by connecting parametric assemblies in grasshopper with a range of different analysis software packages such as ANSYS and SAP2000. This research aims at exploring these tools in order to begin to define explicit territories that they might produce.

It should be noted that this research explicitly focuses on emergent structural morphologies afforded by the defined relationships between interoperable software. This is not to say that other aspects such as social, psychological, economic, or conventionally environmental are not important. Rather, this set of investigations is concerned with providing a base methodology through which other economies and interests could be explored in future work.

2. ENABLING NEW EFFICIENCIES

It should be noted that most, if not all, architectural projections of swarm-generated buildings are imaged as static instances of otherwise dynamic processes. While architects are usually interested in building in general, this raises questions as to what information within the swarm continues to actively contribute to the development of any given project. The fact that a swarm becomes frozen at a single moment of its existence otherwise limits its behavior. However, the complexity of that instance does not dictate a single solution (and in fact resists single solutions), but rather a set of solutions given a set of constraints such as materiality, structural sizing and loading of the structure. To use terminology borrowed from scientific experimentation: by projecting an instance of the swarm as a control variable, other architectural aspects may be explored as dependent variables relative to that form. This research accesses and makes use of the incredible amount of latent information within frozen instances of a swarm in order to discover new structurally informed morphologies.

At first glance, snapping a moment out of an active agent-based space might seem undesirable, it actually opens up an opportunity to overcome (or at least sidestep) a fundamental problem presented by swarms when attempting to integrate them with analytical software. Critical to a swarm's functionality as a design tool is its ability to remain relatively computationally inexpensive. If agents within a swarm have to perform overly complex sets of individual calculations, models either become too slow and inefficient to run given a fixed number of agents or the number of agents must be reduced, thus decreasing the intensive capacity of the model. In other words, a swarm's intelligence is directly related to how many agents it is able to sustain (more is typically better) and how efficiently it can develop solutions (faster is better). Augmenting flows of information should ideally yield more productive results rather than negatively disrupting the otherwise natural circulation of information.

One instance of inefficiency and computational expense within swarms can be found when attempting to form feedback loops between active agent-based models and analytical software, in this case structural analysis (finite element method - FEM) software. The intended purpose of forming such a connection lies in the ability to incorporate structural performance into a swarm's behavior. But structural calculations are computationally expensive as structural sizing, materiality, connections and loading must all be accounted for. This is compounded when dealing with large numbers of structural segments. Perhaps a more critical question lies in what structural logic any instance of a swarm should assume at all. With so many parts and such a range of complex interconnections, a uniform approach would fail to make use of the latent structural intelligence that a swarm has to offer. This is made more difficult given that in many cases, the structural solution is beyond intuitive or conventionally determinable means. By separating structural calculations and spatial swarming from one another, their own efficiencies can be maintained. However, a method for advantageously [re]connecting them without sacrificing efficiency is needed. By making use of Geometry Gym tools in Grasshopper, swarm formations are run through evolutionary feedback loops between Galapagos and SAP2000 FEM software to search for solutions that explicitly express the latent structural intelligence of swarm formations.

3. REAL-TIME INTERACTIVE SWARMS

Flocking swarms are one of the oldest forms of agent-based models. Originally developed by Craig Reynolds (1987), they mimic the spatial behavior of flocks of birds or schools of fish by charging agents with separation, cohesion and alignment values as well as a local envelope of perception. Particle swarm optimization, as described by Kennedy and Eberhardt (1995) is a stochastic global optimization method for continuous functions. This research makes use of a derivative of such a model[i] consisting of spatially located nodes that emit agents which in turn leave traces of their paths as they move through space. The model defines a limit to the length of their paths (in terms of segments) as well as capping the number of agents simultaneously populating the environment (they terminate once they reach their targeted emitter). These agents flock in relation to one another while also being affected by the trails left by previous agents. This allows an agent's history to influence future particles in the system. The resulting swarm serves as an optimal pathfinder through 3-dimensional space given a particular emitter organization (Figure 1).

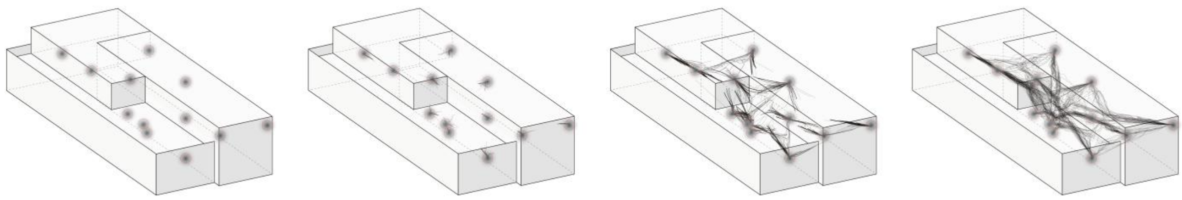

FIG 1 – Emitters within volumetric search space

In an effort to avoid stagnation and provide adjustability, the swarm contains a real-time interface where the user can reposition emitter nodes thus forcing the swarm to reorganize while accounting for the history of the previously optimal organization. The ability to quickly test and augment results allows for the swarm to be easily tuned within a given space. This also provides an opportunity to interactively understand the swarm during states of initialization, equilibrium and transition (Figure 2).

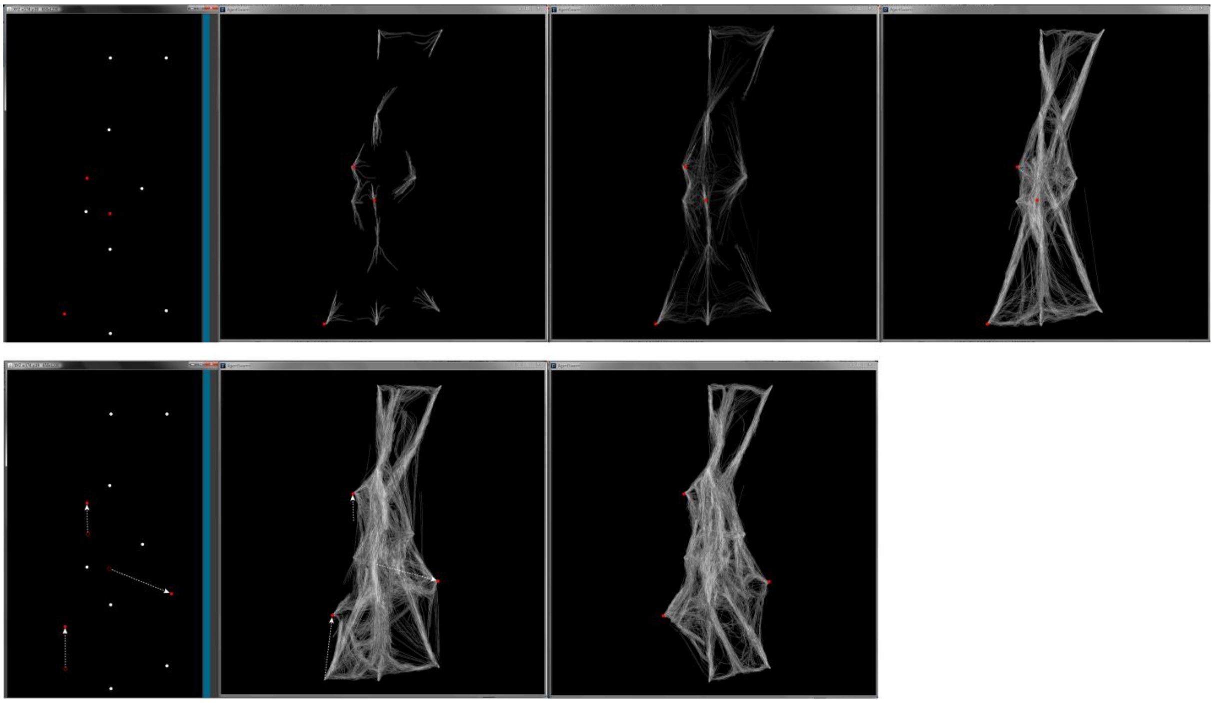

FIG 2: Interactive swarm sequence

4. SWARMED INSTANCES

Any single snapshot of these swarms contain a minimum of several hundred thousand line segments representing each instance of an agent's trajectory through space and as such are quite heavy to process into usable curvature. Simply taking them out of their dynamic environment isn't enough to make them useable within FEM software. In order to manage this problem, these paths are exported, joined, culled and parametrically rebuilt resulting in similar curve networks with a much lighter computational footprint. However, the nature of the flocking swarm does not initially produce a structurally viable formation as most paths tend not to intersect one another. As such, a secondary layer of curves is produced by means of a proximity mesh through the swarm much the same way webbing connects chords to one another within a truss (Figure 3). Alone, the proximity mesh serves as a minimal solution within the excessive redundancy of the swarm. However, the conceptual and aesthetic dissonance between these two layers is seen as undesirable and opens up a new line of questioning that focuses on how the minimal solution of the proximity mesh might be accessed so that the swarm might induce deviations from what otherwise ultimately operates as a single linear span between 2 points.

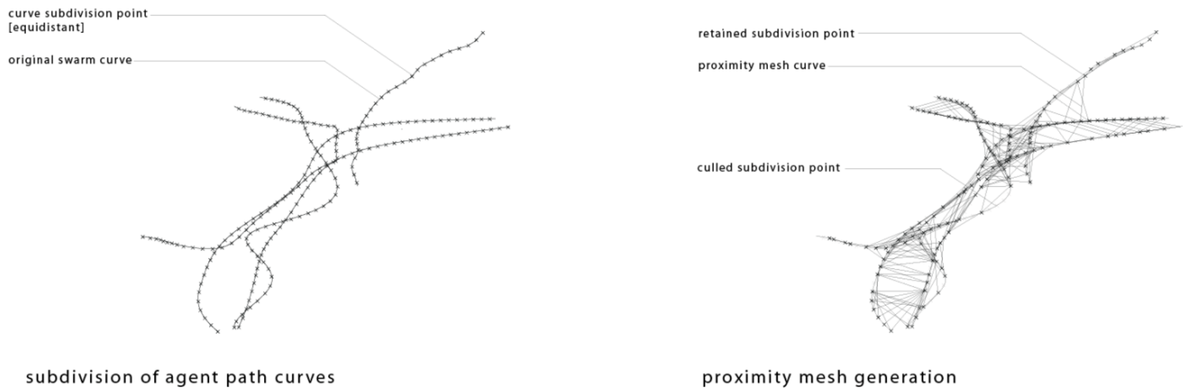

FIG 3: Swarm curve interconnections

5. [RE]MOBILIZING POST-MINIMAL TACTICS

Post-minimalism was at best an incomplete movement discovered only in retrospect by Robert Pincus-Witten defined as “attempts [that] embraced open and unstable modes, forms not only beautiful in themselves despite their unfamiliarity – beautiful on the level of unmediated sensation – but forms that also called into question the stabilized appearance of the day’s abstraction.” (Pincus-Witten, 1987) In many ways, a particular segment of post-minimalist art works similarly to swarms in that minimal sets of common instructions are assigned through large populations of agents such that excessive conditions emerge as a result. Additionally, the ambition of this research is to understand how an isolated (and thus static) form formation might be able to be reactivated, thus overcoming its appearance of stability. The most direct and self-evident instance of this endeavor lies in the suspended fiberglass and string explorations of Eva Hesse (Figures 4a & 4b).

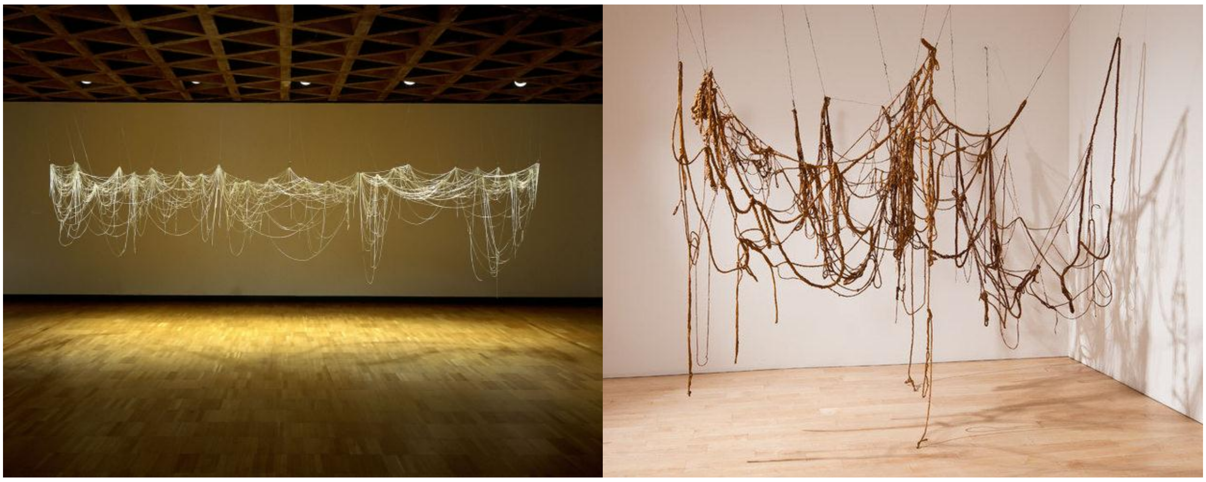

FIG 4: (a) Right After (1969), Milwaukee Art Museum; (b) Untitled (Rope Piece)1970, Whitney Museum

While the conceptual nature of Hesse’s work is akin to the ambitions of these explorations (negative embodiment,[ii] non-signifying form, etc.), the forms themselves also offer literal similarities that the research might borrow from and exploit. First, Hesse’s pieces are a derivative of proximity mesh organizations whereby linear material geometries connect stochastically selected pairs of points in space that fall within a minimum and maximum distance from one another as abstracted in Figure 5.

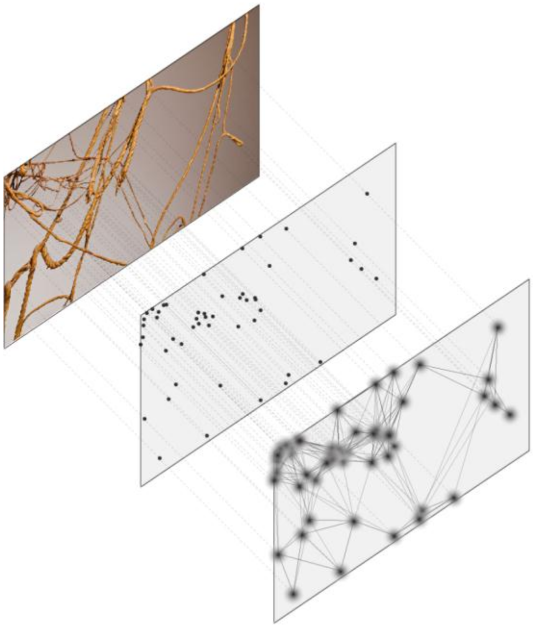

FIG 5: Node projection diagram

This process is repeated in her work with variable lengths of rope until an intensity is achieved capable of effectively expressing the assembly's materially specific intelligence. This similarity opens up an opportunity to understand the straight segments of our proximity mesh as something that can bend and sag in a way that responds to and expresses changing conditions within the swarm. Second, the physical reality of the assembly reveals a set of structural potentials. Immediately obvious to the architect's eye is the prevalence of catenary curves which serve as a diagram of gravity's force through a given length of string fixed between two points. Individually, this might not be particularly interesting, but the aggregate force of multiple catenaries in combination with the suspension cables' connecting nodes in the proximity mesh to the ceiling reveal an indeterminate variability within the assemblage (Figure 6). The shifting of any single node triggers a cascading process of reorganization through the entire system as points shift through space, bending each catenary strand accordingly as it seeks out a new conditional equilibriums. This second instance, while providing insight into how our swarm might behave once physically realized, exposes the necessity to assume that there is not an intuitively obvious equilibrium. Furthermore, changes within a single aspect of the assembly induce changes within the system as a whole within the design process itself. This problem is founded on the high degree of connectivity between parts by distributing the transference of force throughout the assemblage. This realization indicates that our swarm needs a means of evaluating global system performance while incrementally varying and tracking individual components within the assembly in relation to that global performance. We know already from Hesse's work that these relationships are distributed and resist linear performance and results. Similarly a distributed, non-linear method for simulating a form-finding process is needed.

6. STRUCTURAL GRAFTING

Rather than testing these swarms as autonomous, stand-alone objects, a normative structure is used as a host for the PSO assemblies. In order to integrate these two structures into one another, a grafting definition is used so that they can be tested as a whole (Figure 6).

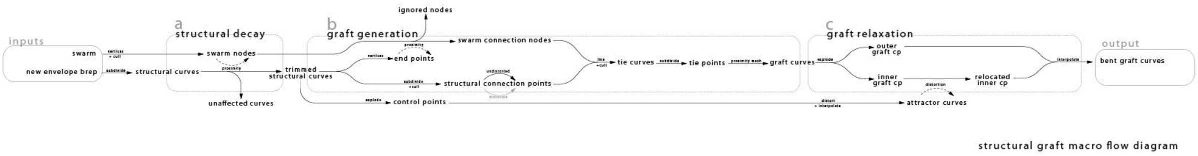

FIG 6: Grafting Macro Flow Diagram

The grafting sequence itself is broken into three parts: (a) structural decay; (b) graft generation; and (c) graft relaxation. Structural decay, as described in Figure 7, operates upon a series of structuralized surfaces that operate somewhere between the massing of the host building and the approximate massing of the PSO formation.

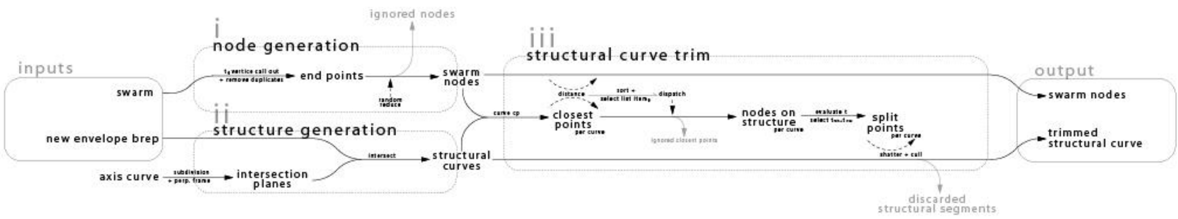

FIG 7: Structural Decay Protocol

Graft generation, as described in Figure 8, produces a series of swarm-like curves that bridge between the edges of the PSO formation and the decayed structure.

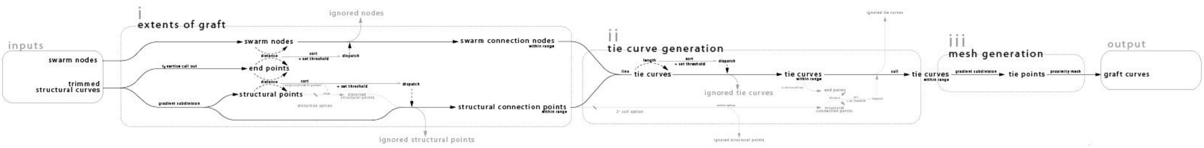

FIG 8: Graft Generation Protocol

Lastly, graft relaxation (Figure 9) takes place which allows the two systems to blend into one another prior to running the system through the evolutionary loop, thus producing an integrated assembly of nodes and structural members that can be generatively sized, modeled, tested and evaluated.

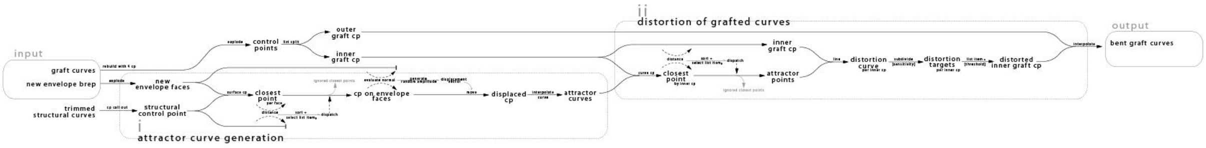

FIG 9: Graft Relaxation Protocol

This process produces a sequence as described in Figure 10.

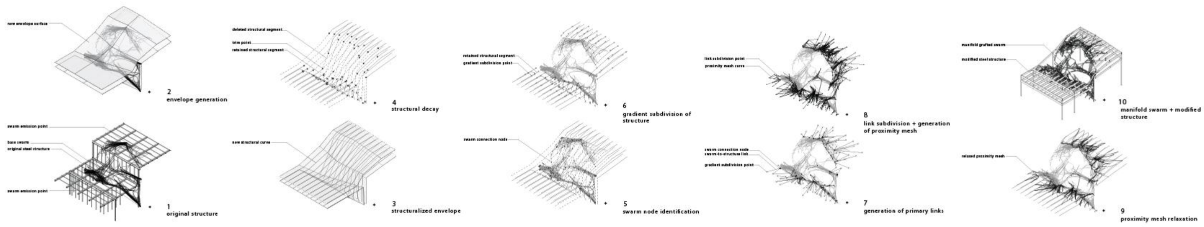

FIG 10: Structural Grafting Sequence

A randomly structurally sized prototype was produced in order to test the aesthetic product as well as preliminarily exploring methods for modeling and rendering a fully skinned instance that covered the normative structure as well as surrounding the grafted PSO structure (Figure 11).

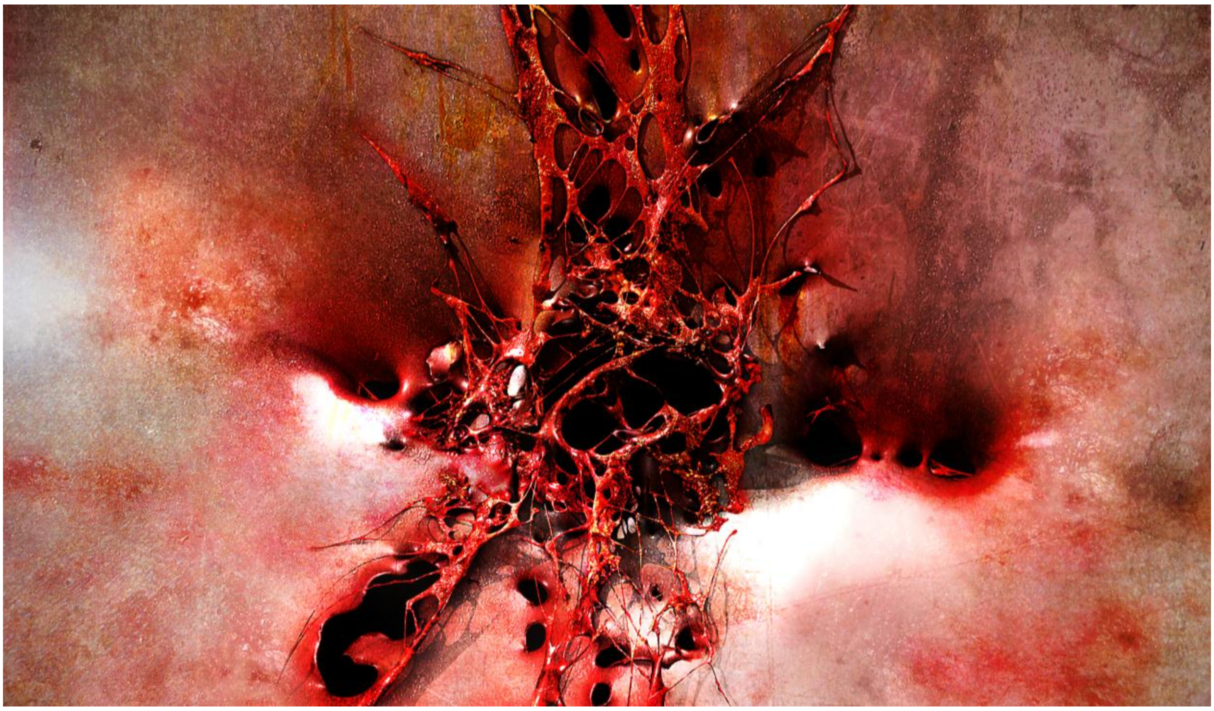

FIG 11: Rendered Structural Graft

7. EVOLUTIONARY MODELING

By establishing a set of variable geometric properties given a singular material,[iii] the swarm can make use of evolutionary processes so that it may arrive at informed structural solutions based on global performance criteria. Toward this end, the swarm is subdivided into sets of component parts forming the variable unit for structural simulation and analysis. It is worth noting that swarms themselves display evolutionary characteristics while they are active. However, once ‘frozen’ they only (and at best) provide a history of that evolution much the same way the rings within a cut tree display a history of a once but no-longer living system.

7.1 Curve and connection point sets

Using a proxy geometry that approximates the swarm components, the geometry is broken down into component parts in order to provide a parametric framework. The first level of subdivision differentiates between agent path curves and proximity mesh curves. A point set is then distributed through each agent path curve based on a fixed frequency of length identifying pinned connection points for the proximity mesh curves (Figure 12). These points are capable of shifting as the assembly deflects. End points of the agent path curves constitute a separate point set that serve as fixed connection points. These points will not shift as the assembly deflects.

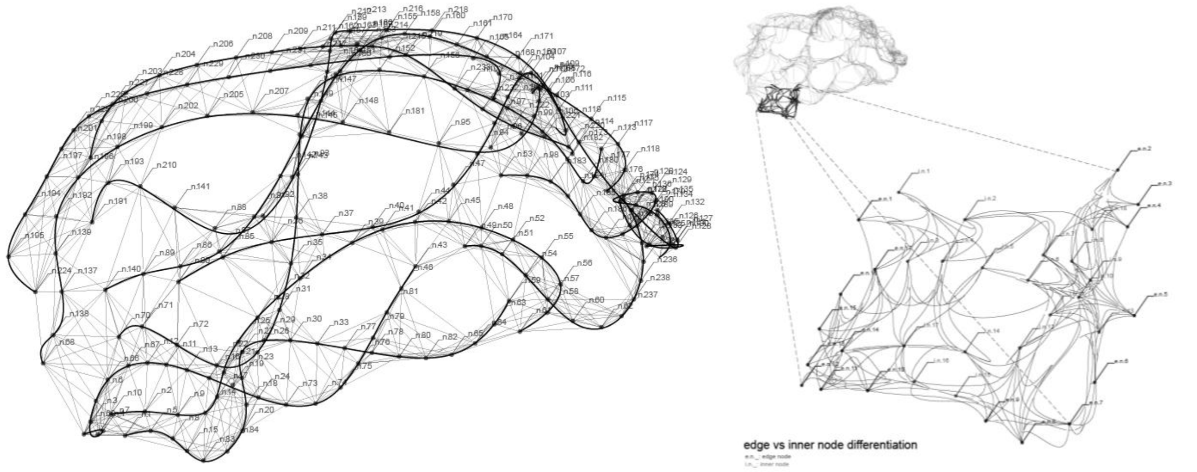

FIG 12: Proxy Swarm Diagram

7.2 Curve Variability

Each agent path curve is assigned a value for structural size with straight segments spanning from point to point within each respective curve. In the interest of limiting evolutionary variability within the high number of proximity mesh segments, curvature, structural sizing, and curve segmentation are compressed into a single parametric component (Figures 13 & 14).

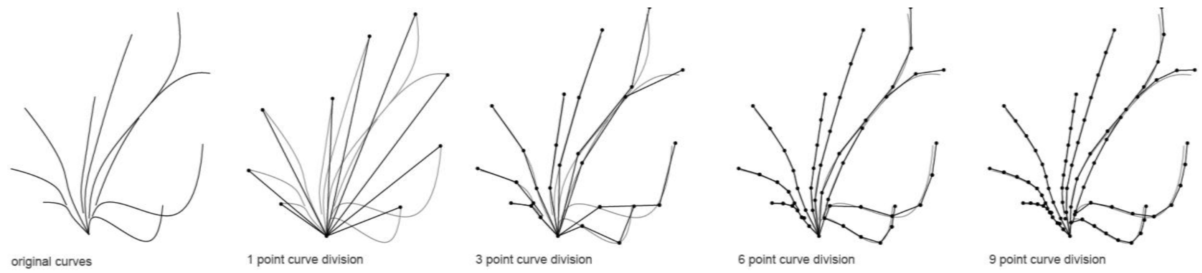

FIG 13: Curve Segmentation Tolerances

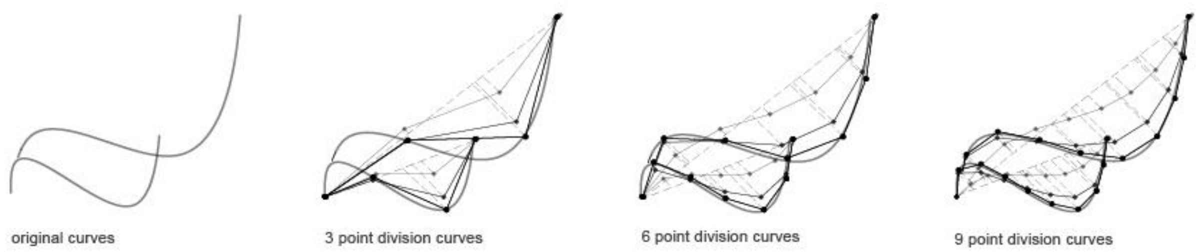

FIG 14: Relative Curve Deflection

The logic of the component is as follows: straight proximity mesh segments are restructured as single span 3-degree curves resulting in a set of 4 control vertices (CV's) per curve. While the end points remain connected to their respective agent path curves, the remaining CV's are each attracted to a nearest point condition found within a differentially scaled set of agent path curves. This process distorts the curves as to provide a way for the swarm, through its internal proximities, to produce locally driven curvature (Figure 15).

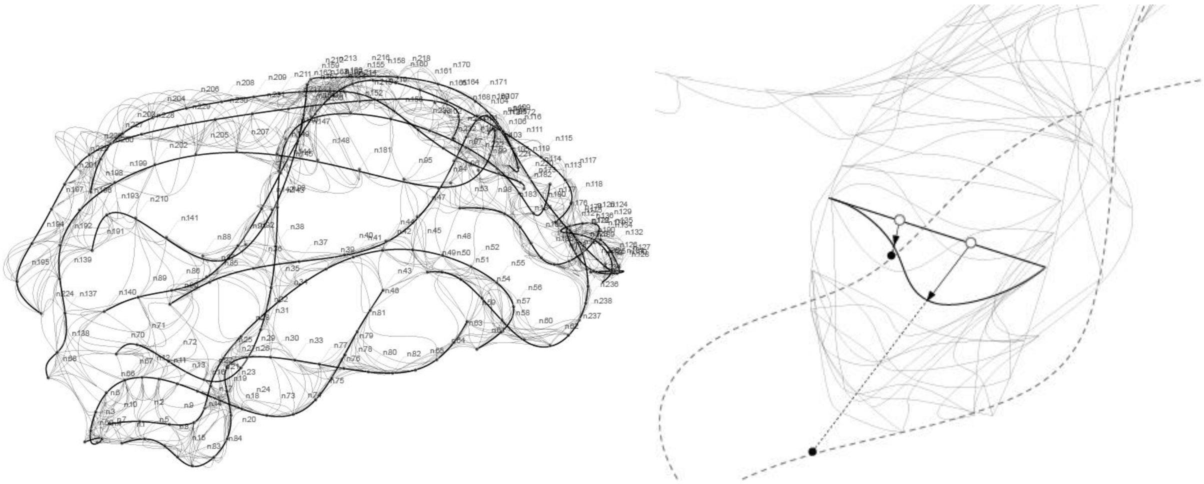

FIG 15: Internal CV Attraction/Deformation

This induces omni-directional curvature that differentiates in magnitude on a per segment basis within the proximity mesh as it distributes through the agent path curves. The relative magnitude of induced proximity mesh curvature is managed through a single sensitivity value. As the sensitivity value increases, so does relative curvature which in turn parametrically drives structural sizing and segmentation.[iv] The combination of agent path curve structural sizing and proximity mesh sensitivity value constitute the variables flowing into the Galapagos evolutionary engine. It is also worth noting that given the scale of internal offset between the two systems with a fixed sensitivity value, a falloff occurs in terms of individual curve length. As the scale increases, fewer and fewer of the longest members are accessed to the full extent of the sensitivity value (Figure 16).

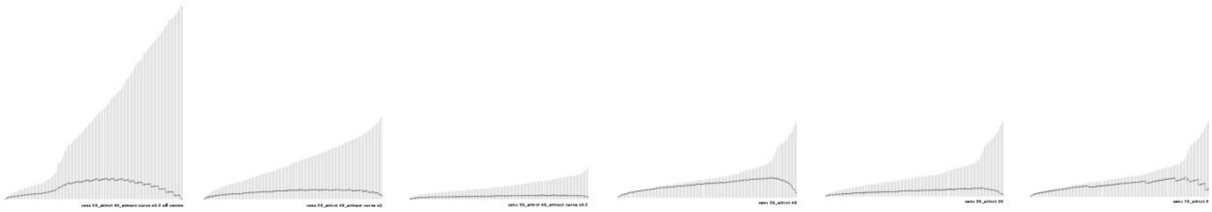

FIG 16: Curve Sensitivity Falloff

7.3 Evolutionary loop

Making use of Geometry Gym tools,[v] these structurally defined swarm assemblies are imported into SAP2000 and tested for deflection at a centralized point given the assembly's dead load. A deflection result is generated for each instance of the assembly and evaluated against a minimal fitness value. By establishing this feedback loop between Galapagos-generated swarm iterations and their resultant deflection values generated in SAP2000, emergent solutions should express the latent structural intelligence of the swarm. The variability provided by the evolutionary engine effectively teases out behavioral tendencies of the assembly in relation to the fitness value provided (Figure 17).

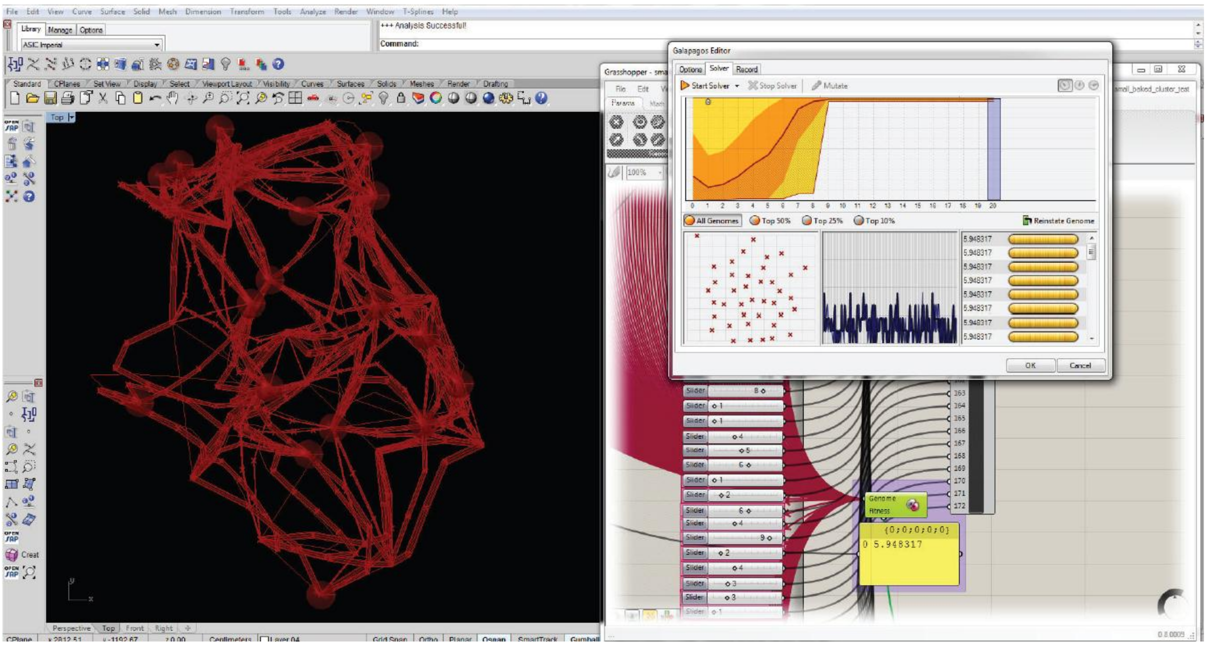

FIG 17: Initial SAP-Galapagos Simulation

8. EMERGENT STRUCTURAL SIZING

The main ambition of this first line of explorations is to assign structural sizing throughout a swarm formation on a per member basis in order to achieve a fitness criteria that is looped through an evolutionary engine. Many of the struggles came with getting the evolutionary engine to approach a solution as some tests did not yield an optimal result (Figures 18 & 19).

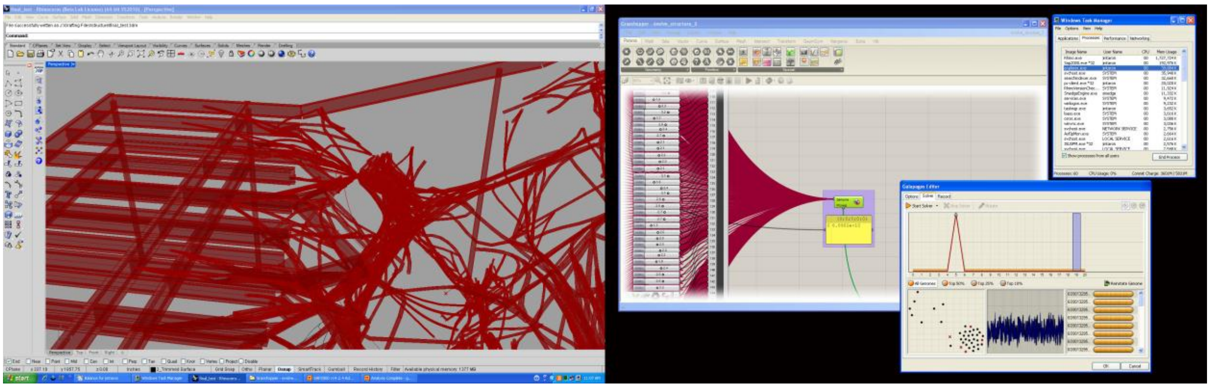

FIG 18: SAP-Galapagos Failed Simulation (screen capture)

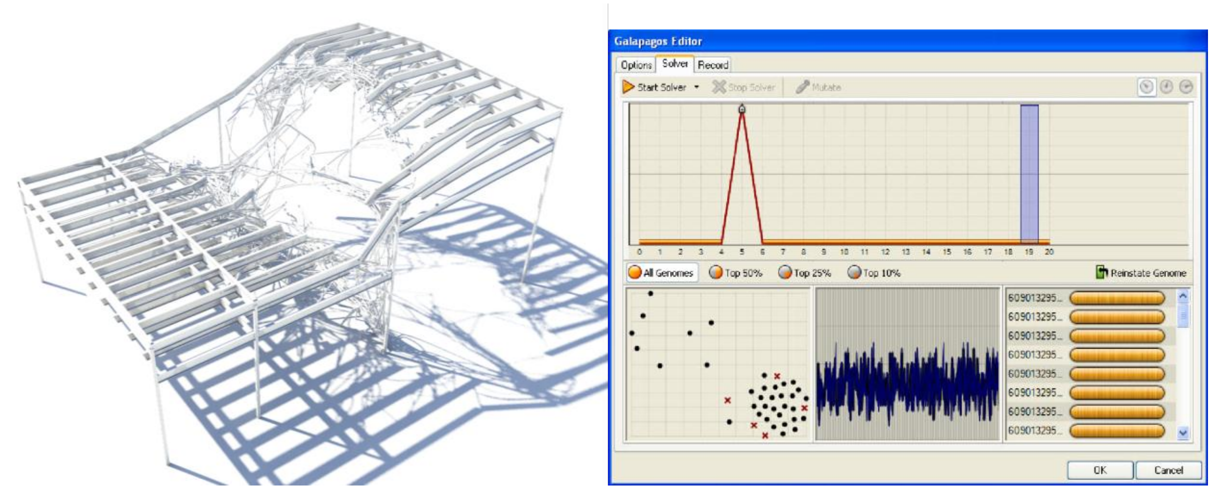

FIG 19: SAP-Galapagos Failed Simulation (detail)

This is attributed to the incomplete nature of the structure itself given that an "incomplete" or not yet optimized instance was sampled. The result is that no solutions were able to approach a fitness value as new combinations of member sizing were not able to generate any evolutionary momentum that value. However, once more robust swarms were tested, optimal assemblies did begin to emerge as evidenced through the Galapagos record (Figure 20).

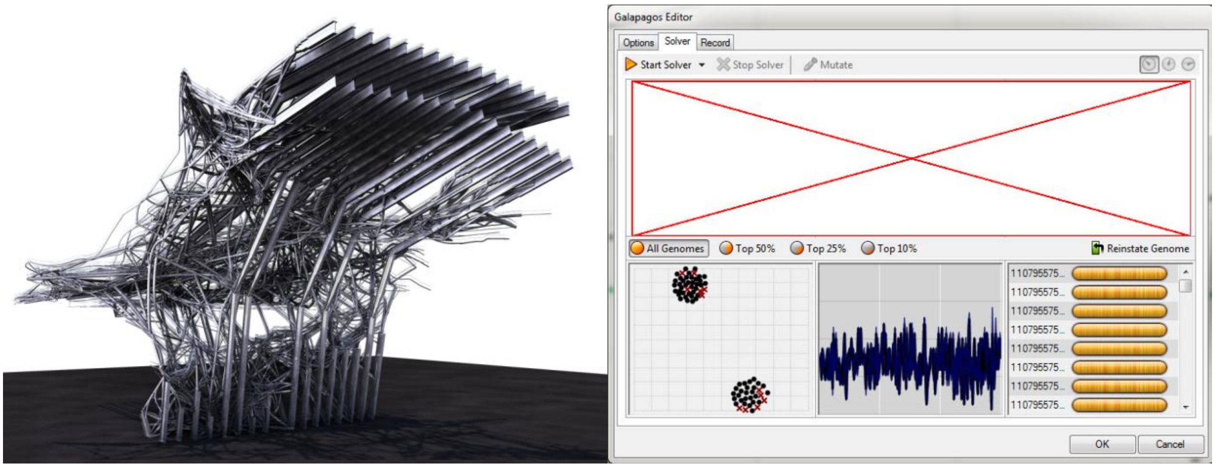

FIG 20: SAP-Galapagos Emergent Solution

Successful instances were then meshed using a meatball definition and very low intensity dynamic mesh relaxation in order to mitigate the unresolved joints between each member as well as testing ideas of skinning the structure itself, similar to how Hesse "skinned" rope with latex. These tests even went so far as to produce a 3D print (Figure 21).

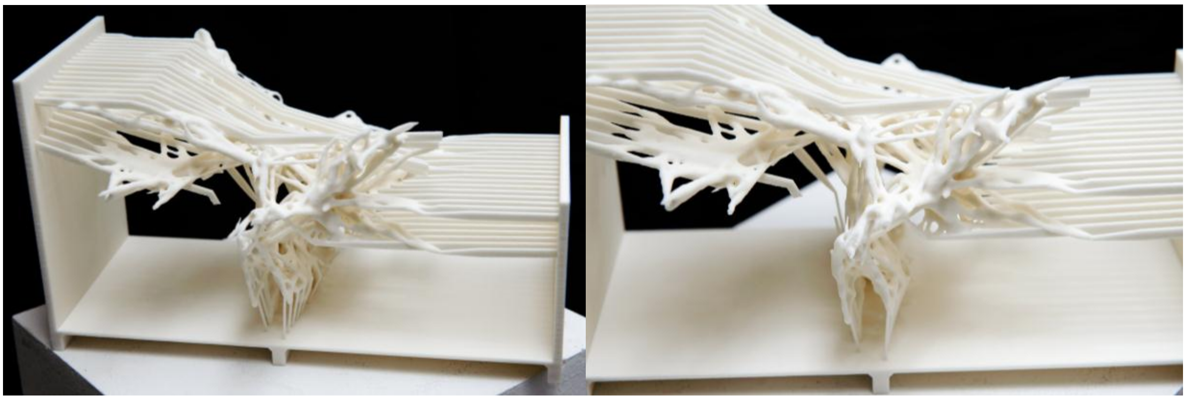

FIG 21: 3D Print

9. BI-DIRECTIONAL STRUCTURAL OPTIMIZATION

While the research presented in the initial issuance of this paper focused solely on individually sizing every member in the swarm assembly, further developments have taken a step back from swarms [before returning to them] in order to articulate the performative advantages of replacing larger scale structural members with swarm-based assemblies that use smaller/lighter structural members. Inspired by Huang and Xie's evolutionary topology optimization of continuum structures that use displacement constraints [Huang and Xie 2008], we began by developing a parametric definition that would allow a single span beam to be evaluated for a specified deflection value (SDV) through SAP2000. By minimizing the absolute value between the deflection result and a SDV in Grasshopper, evolutionary iterations through Galapagos would yield a specific structural size that would approach the SDV as shown in Figures 22 and 23.

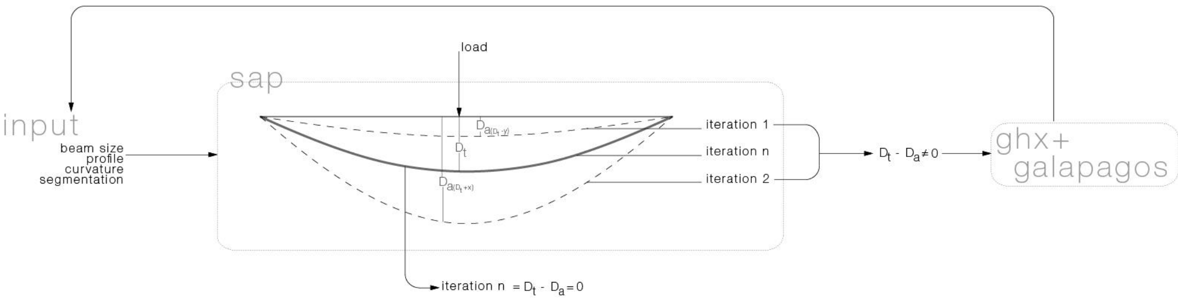

FIG 22: Specified Deflection Diagram

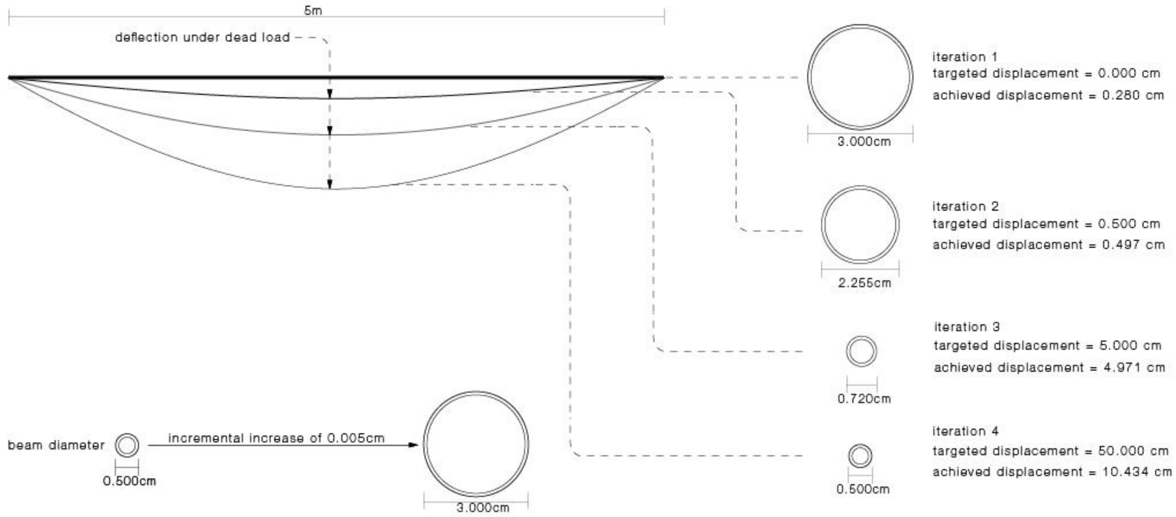

FIG 23: Specified Deflection Emergent Variations

Having established the necessary computational framework for our own evolutionary structural optimization, we began applying swarm assemblies in place of the singular structural member in an attempt to drive the size of the structural members down while maintaining the ability to achieve a range of SDV's given an axial load of 1kN to put the assembly into compression. All members in the assemblies were assigned a uniform value for size and employed a simple proximity mesh to constrain the otherwise disconnected swarm courses to one another. Initial results are shown in Figure 24.

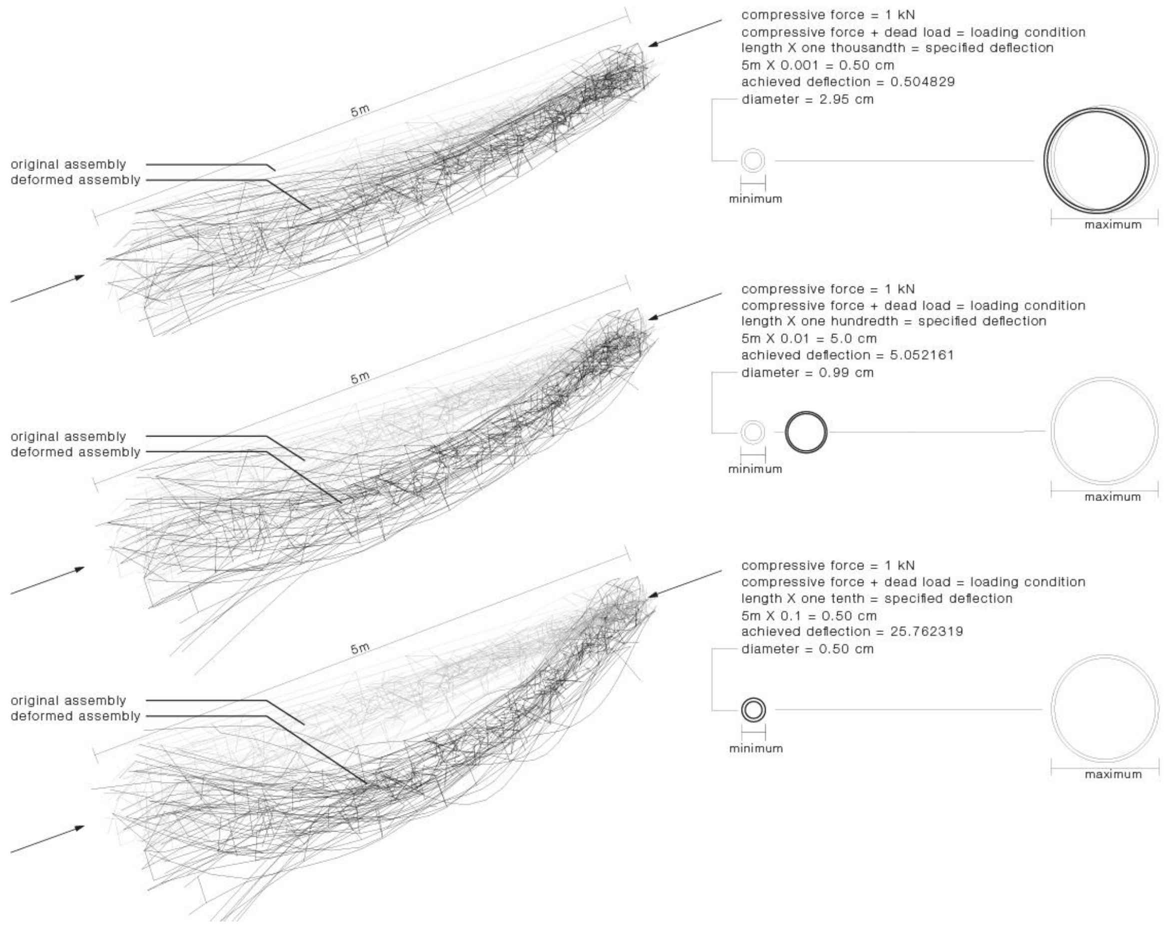

FIG 24: Initial Results from SDV Swarm Variations

The results of these tests exibit a desired correlation between lighter members and higher SDV. However, the tests raised questions over the intensity of the proximity mesh and the effect they were having over achieving deflection. Toward this end, the proximity mesh was sorted into a list that measured their lengths and put them into sets representing increments of 25% of the total population. The tests were run again to achieve the specified deflection values of .001, .01 and .1 of the overall beam span. In this series the population of connecting members was culled by 25% increments beginning with the longest members so as to understand the effects of decreasing structural frequency. Figure 25 documents the results from the .01 deflection value. Please note that results that used only the shortest 25% of the population failed to manage the axial loads and as such they are not shown.

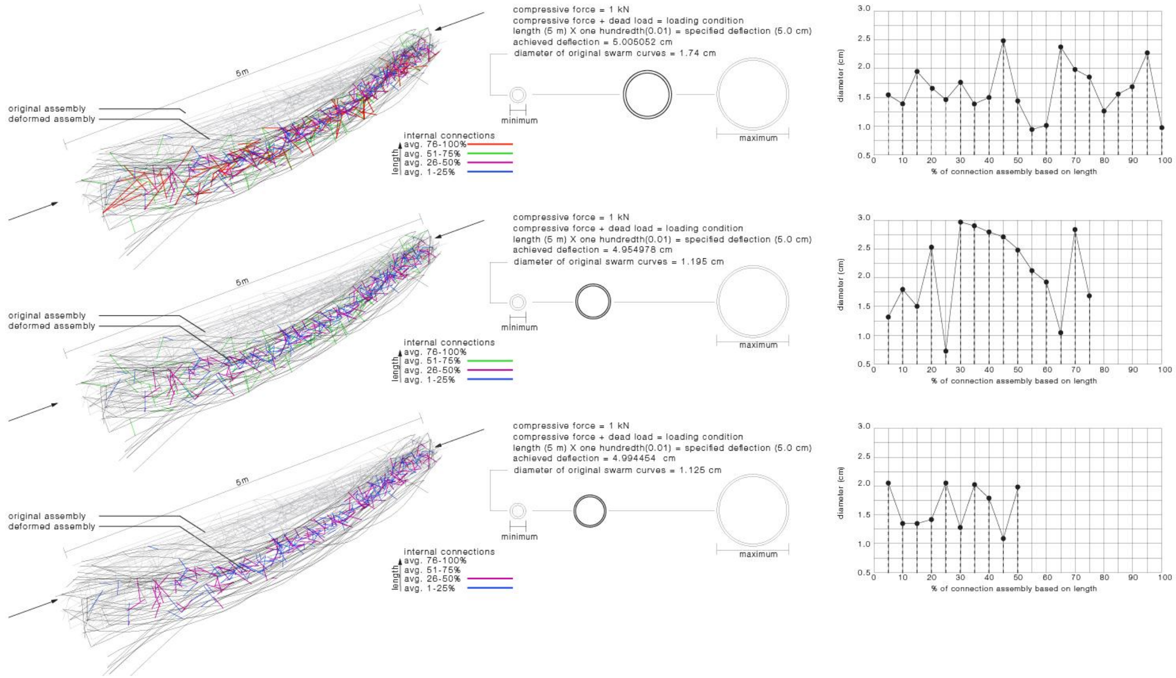

FIG 25: Culled SDV Results

Two unexpected behaviors were expressed in these tests. First, even with the full population of proximity mesh structural segments in place, the longest members contained the highest stress loads. This was surprising in that we expected stress either to appear toward the middle of the span or in areas where other connections were not being made. Second, the test demonstrated the intelligence to size up the structural members in order to achieve a deflection value that had previously relied on more parts throughout the assembly. We were not surprised to see shorter members demonstrate higher stress levels given the absence of additional structural members.

With these results in hand, the grafting definition (described in section 6) was modified so that as additional branches were added to the assembly, they became shorter given each successive generation in an effort to geometrically transition from long-span swarm courses to the smooth surface of the wall. Using this in combination with the culled members establishes a bi-directional evolutionary modeler. Tests were initiated on a single member that underwent 2 orders of branching the curve parameter and angle of differentiation were given a limited possible range and then randomly selected. The branching definition was then evaluated in terms of deflection and structural sizing as to try to understand the local effects in terms similar to the previous iterations as shown in Figure 26.

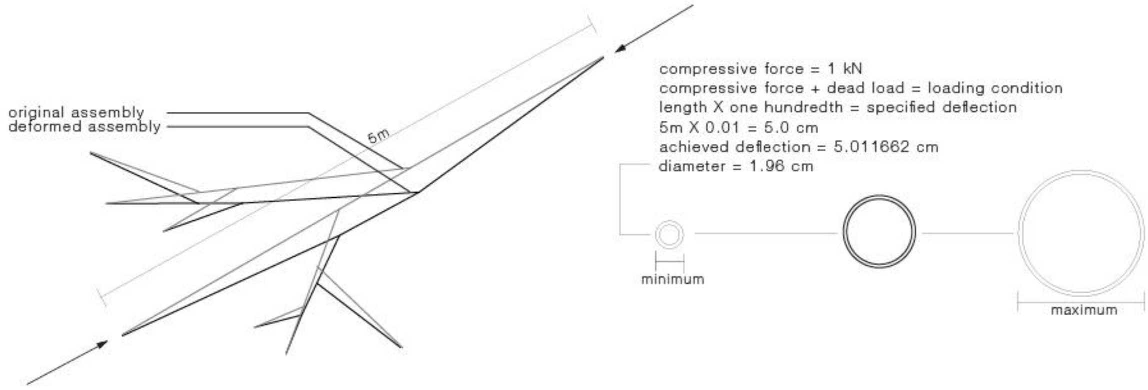

FIG 26: SDV Grafting Branch Sequence

This method has application in adaptive reuse of existing structures whereby swarmed geometry erodes initially existing structure. Bi-directional, stress-based grafting then allows the new form to achieve integrative structural viability between the 'new' and 'old' structures as shown in Figure 27.

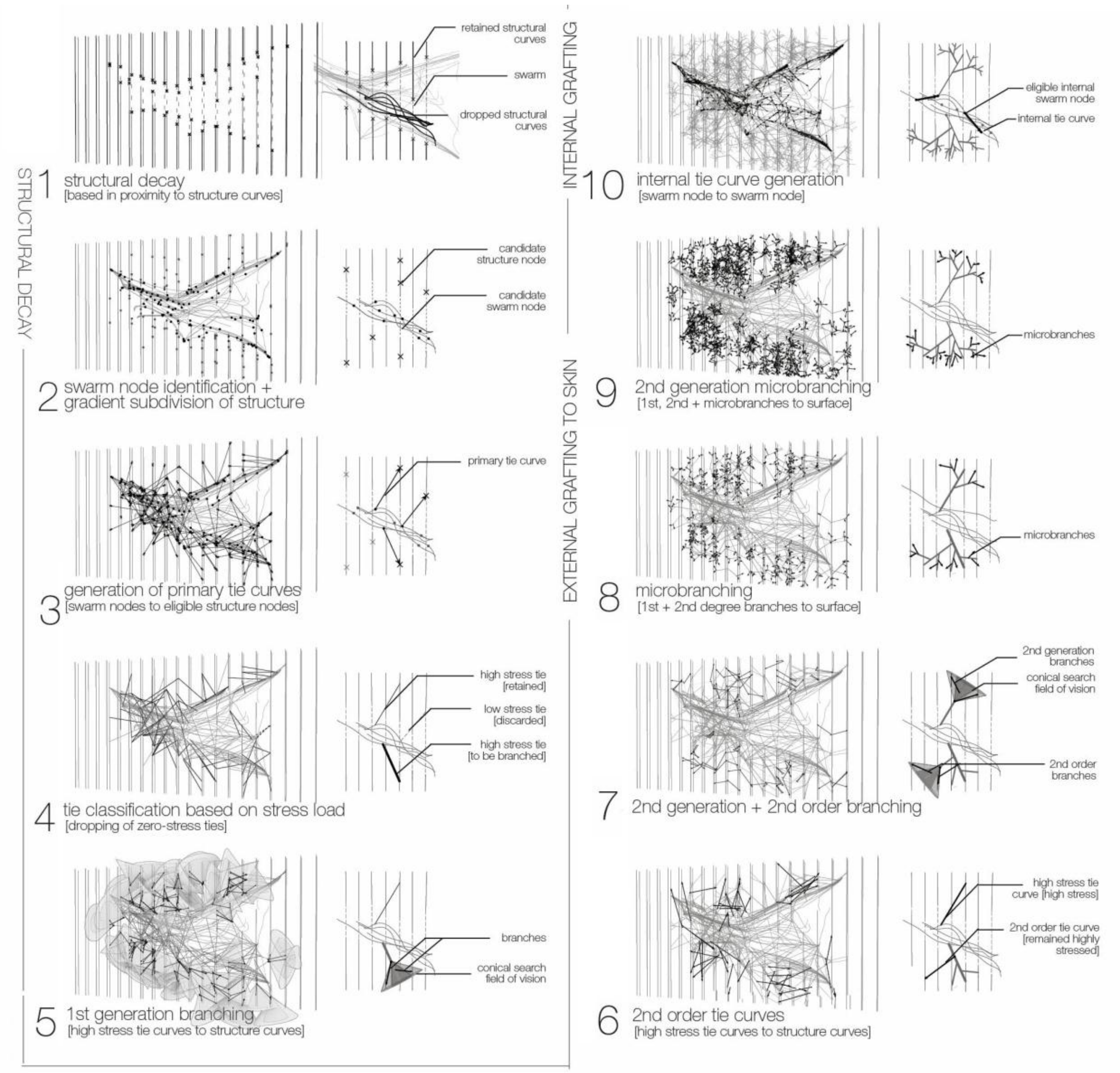

FIG 27: Bi-Directional Grafting Sequence

The resulting mesh geometry (accounting for the entire integrated form) was dispatched into three stress categories and assigned different materials; high stress facets were assigned stiffer material while lower stress facets were assigned more flexible material as shown in Figure 28.

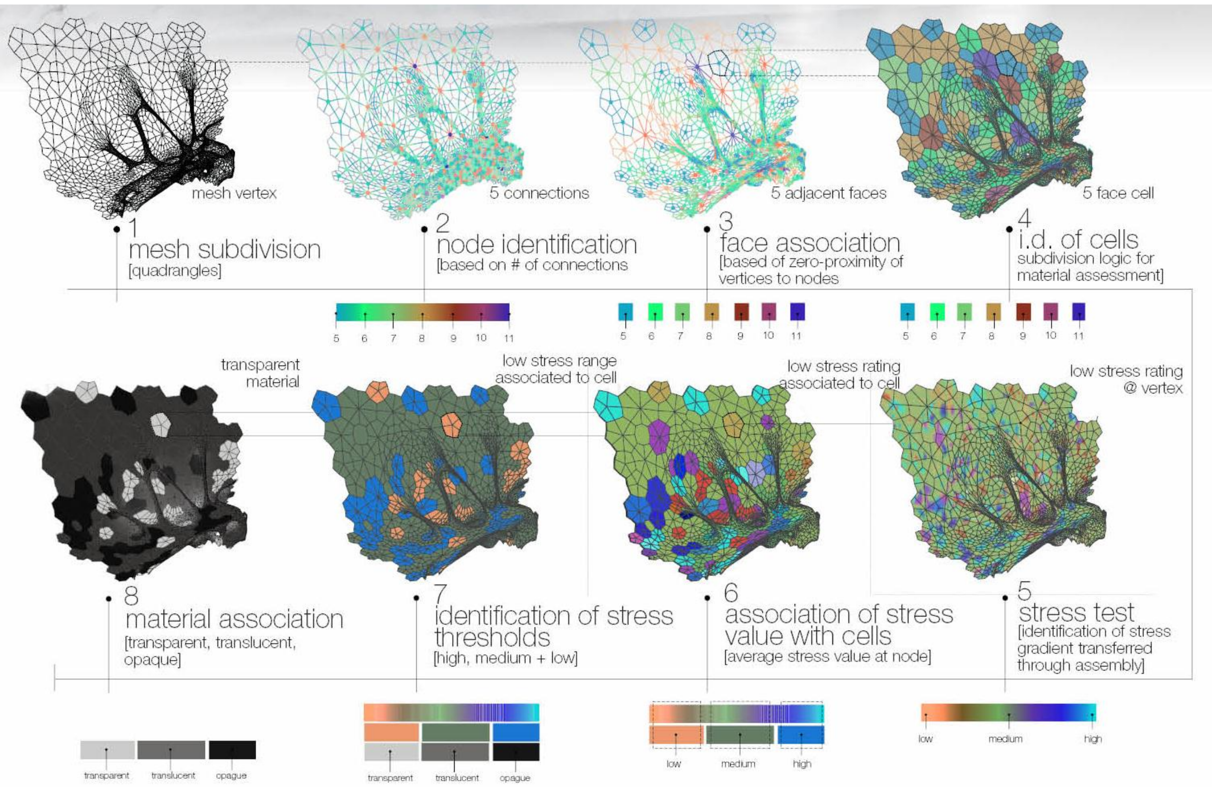

FIG 28: Stress-generated Material Assignment Diagrams

While the aesthetic impacts of such a system are immediately evident (Figure 29), what is more important to note is the structural sensitivity and material efficiency that these processes yield when considering the adaptive reuse/augmentation of existing structures.

FIG 29: Project Render

10. CONCLUSIONS AND FUTURE WORK

The methods presented in this paper represent two and a half generations of investigations with structurally intelligent swarms that range from random evolutionary variation measured to specified performance criteria. The results have been promising but have a great deal of room for future work - most specifically in the area of full scale fabrication. Additionally, the techniques aim to partner with urban analysis that both quantifies and identifies the volume of potentially reusable space in Calgary that might benefit from Swarm Intelligent Systems (SIS) augmentation. This will not only broaden the range of applications for SIS but also generate real-world instances that go beyond conceptual and computational projections. The next generation of projects coming out of Synthetiques and the Laboratory for Integrative Design aims at continued use of SIS methods at various scales. Both the premise of structurally informed swarms and explicit techniques for grafting into existing structures is an exciting one that promises a robust future in the context of efficiency, adaptive reuse and sustainability as we seek out new ways to inhabit and re-inhabit the polis.

11. ACKNOWLEDGEMENTS

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[i] This is a modified version of a piece of open-source Processing code written by Ioannis (Yiannis) Chatzikonstantinou (http://prototy.blogspot.com)

[ii] For a remarkably insightful view into this concept, please see Denise Birkhofer's article, "Eva Hesse and Mira Schendel: Voiding the Body — Embodying the Void" in Women's Art Journal, Fall/Winter 2010.

[iii] In this case, round steel tubing is chosen in anticipating of the assemblies might be able to locally replace normative steel assemblies in existing buildings capable of omni-directional connections.

[iv] Original tests decreased structural sizing as curvature increased while a second set of testing inverted this relationship whereby structural size increased as curvature increased.

[v] These tools include Structural Draw, SSI Sap and SSI SAP Solver. Downloads available at http://geometrygym.blogspot.com.