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# SCHEDULE COMPRESSION USING FUZZY SET THEORY AND CONTRACTORS JUDGMENT

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***SUMMARY*:** This paper presents a new method developed for schedule compression of non-repetitive construction projects. The method accounts for uncertainties associated with crash cost and it considers contractors' judgment. It allows contractors to: (1) perform risk analysis for different schedule compression plans; and (2) perform different scenarios expressing vagueness and imprecision of estimated crash cost using a set of measures and indices.The method combines Fuzzy Set Theory and contractors' judgment in setting priorities for the compression process of project schedules. The developed method is implemented in MS-Excel, and it can be easily used as an add-on utility to other scheduling software systems. To illustrate its capabilities, an example project drawn from the literature was analyzed.

***KEYWORDS:*** schedule compression, fuzzy sets, non-repetitive construction, risk, uncertainty

***REFERENCE:*** *Osama Moselhi, Adel Alshibani (2013) Schedule compression using fuzzy set theory and contractors judgment, Journal of Information Technology in Construction (ITcon), Vol. 18, pg. 64-75, http://www.itcon.org/2013/4*



## 1. INTRODUCTION

Schedule compression is a challenging task, which project teams frequently face when there is a need to reduce durations of projects in an effort to meet contractual obligations, changing client needs, recover from delays experienced during project execution and/or to determine least cost project duration. The main challenge here is to reduce project duration with the least amount of extra cost (Moselhi and Alshibani, 2011). Schedule compression involves uncertainties which may arise from (Khodakarami et al., 2007): (1) uniqueness of projects; (2) variability (trade-off between performance measures like time, cost and quality); and (3) ambiguity (lack of clarity, lack of data, lack of structure and bias in estimates). Schedule compression is a process that requires additional resources. Therefore the risk associated with this process has to be taken into account (Shankar et al., 2011). The literature reveals that considerable efforts were made in developing methods for schedule compression using different techniques: (1) heuristic procedures (Siemens, 1971; Moselhi, 1993); (2) mathematical programming (Henderickson, 1989; Pagnoni, 1990); (3) computer simulation (Wan, 1994); (4) simulation and genetic algorithms (Wei Feng et al., 2000; Ding, 2010; Zheng et al., 2004), (5) genetic algorithms and fuzzy set theory (Eshtehardian et al., 2008a; Eshtehardian et al., 2008b).

The main setback of heuristic-based methods is that the performance is problem-dependent, and good solutions are not guaranteed (Wei Feng et al., 2000; Ammar, 2010). As to mathematical programming, the main limitation is the excessive computational effort in view of the number of options to complete an activity and the large number of activities in actual projects. Computer simulation based methods, on the other hand, require dedicated simulation professionals (Hajjar and AbouRizk, 2002) and expert opinions in absence of numeric data (Chung, 2007). Detailed descriptions of the main shortcomings of simulation were highlighted in literature (Hajjar and AbouRizk, 2002; Chung, 2007; Ferson, 2002; Shaheen et al., 2007). Also in simulation, the causal relationship between sources of uncertainty and project parameters is not modeled (Khodakarami et al., 2007).

An alternative approach to deterministic and probabilistic methods is fuzzy set theory (FST). Fuzzy set theory, pioneered by Zadeh (1978), is useful for representing and modeling uncertainties, particularly in absence of historical data. Compared to simulation, modeling uncertainty using fuzzy set theory is computationally simpler, not very sensitive to moderate changes in the shapes of input distributions, and does not require the analyst to assume particular correlations among inputs (Shaheen et al., 2007). Fuzzy set theory has been used in the development of many applications in construction engineering including; pricing construction risk (Paek, 1993); project network schedule (Lorterapong and Moselhi, 1996); reliability assessment (Booker and Singpurwalla, 2002); and range cost estimating (Shaheen et al., 2007). As to schedule compression models, fuzzy set theory is used along with simulation and/or genetic algorithm (e.g., Zheng and Thomas Ng, 2005; Eshtehardian, 2008a; Eshtehardian, 2008 b). The later methods consider only cost in the schedule compression process. This paper presents a multi-objective method to circumvent the above stated limitations in schedule compression of construction projects. The developed method accounts for cost, contractors' judgment and for the uncertainties associated with the direct cost of crashing activity durations. The use of FST, as presented in this paper, is particularly suited for the problem at hand due to two main reasons. Firstly, crashing durations of project activities is frequently carried out during construction subjected to the unique conditions of each project environment. As such, there is no historical data for each and every activity being considered for crashing. This is contrary to cost planning of the original projects' scope of work prior to construction; where historical data may exist to support the use of probabilistic methods. Secondly, FST facilitates the direct utilization of expert knowledge that applies to the unique conditions of each project at hand through the use of membership functions that best suit these unique conditions.

## 2. PROPOSED METHOD

The proposed method integrates contractors' judgment and crashing cost in setting up priorities for crashing activities on each critical path in projects' network schedules. Contractors' judgement accounts for their experience, while crashing cost accounts for the risk associated with the additional direct cost needed to compress critical activities. The priority assigned to each activity on the critical path is a joint priority; calculated using judgment-based priority and cost-based priority. Relative weights are assigned to contractors' judgment and crashing cost. The risk associated with crashing cost is modeled using fuzzy set theory. The developed method is automated in a computer application developed in MS-Excel, which interacts with Microsoft Project 2010 during the schedule compression process. Microsoft project is used to perform critical path method (CPM) analysis, which identifies critical path(s) of the project under consideration while the developed MS-Excel application is used to carry out the required calculations of the proposed method. Based on fuzzy set theory, crashing costs of critical activities are modeled by fuzzy numbers as described subsequently.

In view of the constraints and unique conditions associated with crashing activity durations to accelerate project delivery, it was deemed helpful to account for risk and uncertainties in performing the compression process. The developed method is applicable during the execution phase of the project, i.e., after the contract has been signed and commencement of construction on jobsite. As such, normal cost and normal duration of project activities are considered to have crisp value as stipulated in the contract documents. The main components and the sequential operations in the computational process of the developed method are shown in Fig. 1. The steps of computations required for the application of the proposed method can be summarized as follows:

1) Perform CPM analysis of the project under consideration using Microsoft Project (or any scheduling software that can identify the critical path(s)).

2) Export the generated schedule data to the developed MS-Excel application.

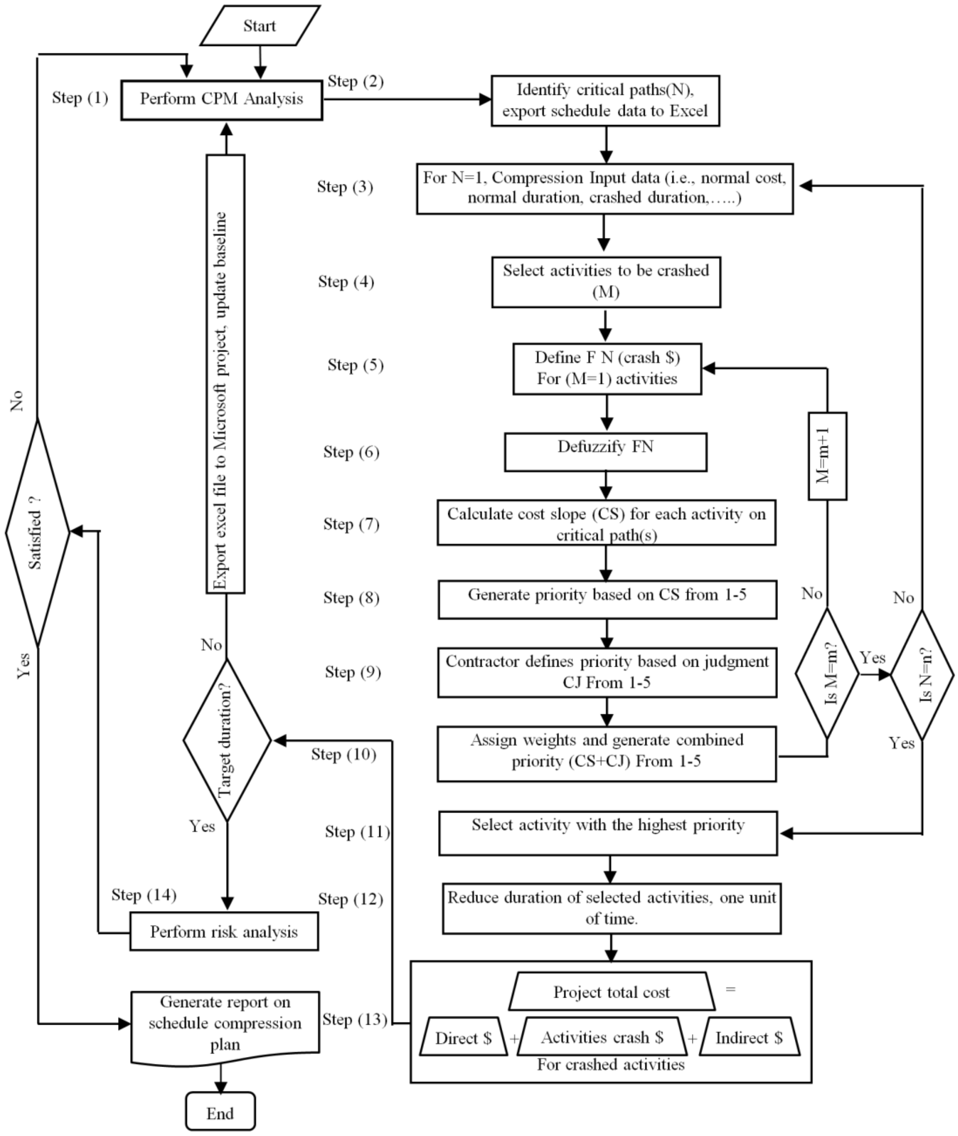


*FIG. 1*: *The flow chart of the proposed method*

3) Identify the number of existing critical path(s) “N”, and the number of activities on the identified critical path(s) “M”. Input the compression data for activities on the generated critical path(s) in the MS-Excel application by the user. This data includes normal cost, normal duration and crashed duration for each critical activity.
4) Model uncertainties associated with crash cost of activities using fuzzy numbers similar to that shown in Fig. 2. It should be noted that “a” and “d” are the lower and upper bounds of the estimated crashed cost, which have membership f(x) = 0.0 , while “b” and “c” are the lower and upper modal values of the estimated crashed cost, which have full membership (i.e., f(x) = 1.0).

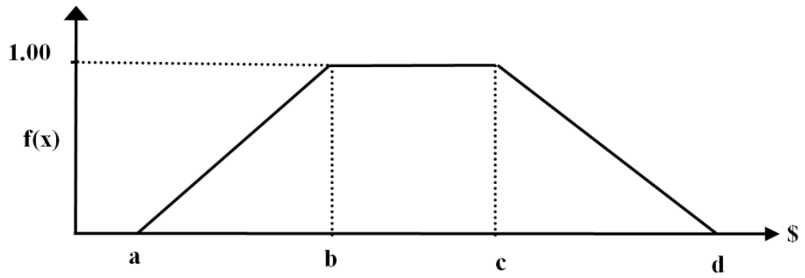


*FIG. 2: Trapezoidal fuzzy number*

5) Defuzzify the fuzzy estimate defined in Step 3 using the centre of area (COA) method (Shaheen et al., 2007), which represents the expected value using Equation (1):

$$\text{EV Trapezoidal} = a + \frac{2(c-b)(b-a) + (b-c)^2 + (b-a)(d-a) + (c-b)(d-a) + (d-a)^2}{3(c-b+d-a)} \qquad (1)$$

6) Calculate and tabulate the "Cost Slope" of activities on the critical path(s) knowing the expected value of the crashed cost and the activity crashed duration as shown in Fig. 3 (A).

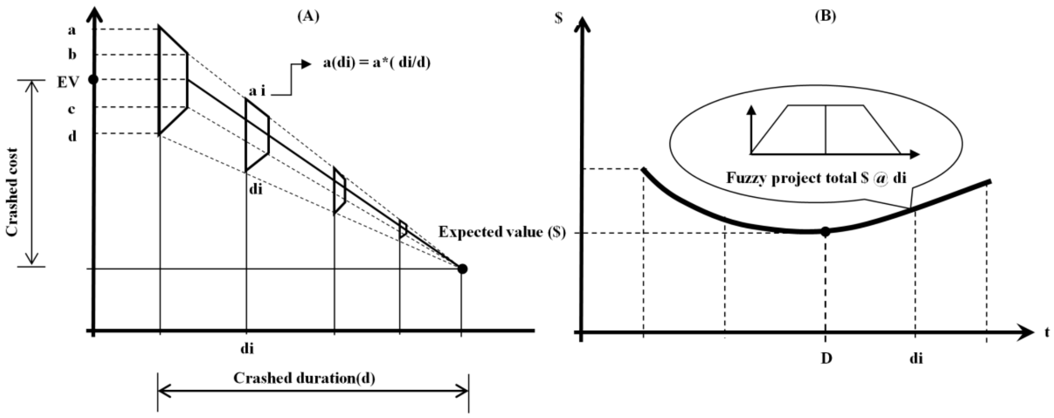


*FIG. 3: (A) Fuzzy crashed direct cost, (B) Time-Cost Trade-Off*

7) Assign priority ( $P_{CSi}$ ) for each activity on each critical path based on their respective cost slope calculated in Step 6 above. The activity with the least cost slope is assigned a priority of 5 and the activity with the highest cost slope is assigned a priority of 1 and the rest can be assigned accordingly.
8) Define priority based on contractor's judgment ( $P_{CJi}$ ), also, on a scale from 1 to 5. These priorities can account for his experience and preference, such as crashing early activities rather than those that will be performed later, those that are less risky or those that have their needed recourses in-place.
9) Assign relative weights and generate joined priority ( $P_{CSi} + P_{CJi}$ ), using Equation (2) below:

$$PI_i = P_{CSi} \times W_{CS} + P_{CJi} \times W_{CJ} \qquad (2)$$

In which;

$P_{CSi}$ is the priority assigned to activity based on the cost to compress or crash the $i^{th}$ activity one unit time;

$P_{CJi}$ the priority assigned to activity i based on the contractor's experience and judgment. $W_{CS}$ , $W_{CJ}$ are the weights assigned to cost and contractor experience, respectively.

10) Crash first the activity that has the highest combined priority. If more than one activity have the same combined priority, then crash the one that exists on more than one critical path. If not, crash the activity that finishes earlier. In the event of having more than one critical path with no common activities, simultaneously crash the activity that has the highest priority on each path.
11) Calculate the project fuzzy total cost at each crashed duration using Equation (3) below. Fig. 4 illustrates the mathematical calculation of project total fuzzy cost. As can be seen from Fig. 3(B), the project fuzzy total cost at any time (di) is calculated using the following equation:

$$FTC = [a,a,a,a]_{Pdc} + [a1_{pic}, a1_{pic}, a1p_{pic}, a1_{pic}]_{di} + \sum_{i=1}^{n}([a_{cc}, b_{cc}, c_{cc}, d_{cc}]_{di})_i \qquad (3)$$

In which;

$FTC$ is project fuzzy total cost at (di);

$a_{pdc}$ is project fuzzy direct cost;

$a1_{pic}$ is project fuzzy indirect cost at (di);

$a_{cc}$ is fuzzy crash cost of activity(i) at time (di).

$i$ is number of activities to be crashed at time (di);

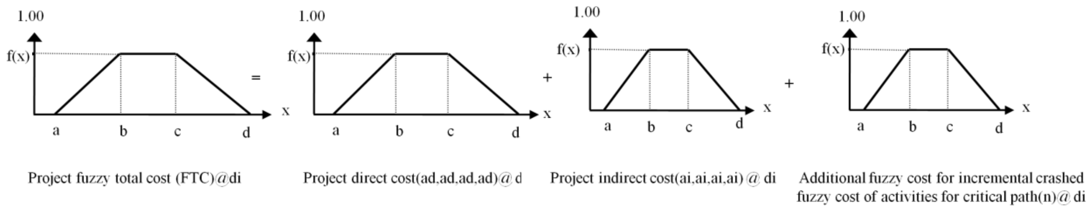


*FIG. 4: Calculation of fuzzy total cost*

12) Repeat the above steps until the target project duration is met, the least cost project duration is found or until no further crashing is possible.
13) Record at each increment of time reduction, project direct, indirect and total costs and the associated duration. Plot project cost against its duration.
14) Analyze the generated output, and perform risk analysis for the cost of the selected schedule compression plan using the indices and measures described below (see also the calculations and analyses presented in the numerical example).

## 3. INTERPRETATION OF FUZZY OUTPUTS

A number of measures and indices were introduced to interpret the results obtained based on fuzzy set theory. The possibility measure (PM), as introduced by Zadeh (1978), intends to evaluate the degree of belonging to the membership of a fuzzy number. According to Kaufmann and Gupta (1985), the PM is the law of possibility which is a unique concept in fuzzy set theory, and it can be applied to evaluate the occurrence possibility of different events. The most possible and plausible variable in a fuzzy number is the one that has a possibility measure of 1.0; i.e., has a membership value of 1.0. In the proposed method, the possibility measure is applied to evaluate the possibility of achieving a certain cost for each selected schedule compression plan. It is also applied to evaluate the possibility of having the cost of a compression plan falling within a defined range or being at a given crisp value, as described later in the project example. It should be noted that in applying the possibility measure no consideration is given to the size of the intersection area generated from the membership function that represents the tested event and the function that represents the targeted event (see Fig. 6). This may lead to high possibility value while the intersection area is small. The possibility measure in certain circumstances does not provide an insightful assessment of the compatibility between the two fuzzy events referred to earlier (Lorterapong and Moselhi, 1996).

The agreement Index (AI), which was introduced by Kaufmann and Gupta (1985), on the other hand, can be used to compliment the possibility measure. The agreement index measures the ratio of the intersection area between the two fuzzy events with respect to the area of the event being assessed. For example, assuming that A and B are two events, the agreement index of A with respect to B; AI (A, B) is defined as:

$$AI(A,B) = \frac{area\ A \cap B}{area\ A} \tag{4}$$

The area of intersection can be determined from partial integration given the four numerical values for a trapezoidal fuzzy number [a,b,c,d]. Also, fuzziness (F) and ambiguity (AG) measures were introduced to describe vagueness and lack of precision, respectively. The fuzziness measure (F(A)) used in this paper is based on that developed by Klir and Folger (1988), and it can be calculated using the following equation:

$$\mathrm{F(A)} = \int_a^b (1 - |2A(x) - 1|\, dx = b - a - \int_a^b |2A(x) - 1|\, dx \tag{5}$$

The ambiguity measure can be calculated as follows (Shaheen et al., 2007):

$$\mathrm{AG(\mu)Trapezoidal} = (c-b)/2 + [(d-c) + (b-a)]/6 \tag{6}$$

For a crisp number and a fuzzy uniform number, the fuzziness measure equals zero because the lack of distinction between a fuzzy uniform number or a crisp number and their complements is zero. The variance of fuzzy numbers introduced by van Dorp and Kotz (2003) is used in the developed method to provide a measure of how far from the expected value the tested numbers lie. The variance of a trapezoidal fuzzy number (a,b,c,d) is calculated as:

$$\mathrm{Variance(Trapezoidal)} = \left( \begin{array}{l} \frac{(b-a)}{(d+c-b-a)}\left(\frac{1}{6}(a+b)^2 + \frac{1}{3}b^2\right) + \frac{1}{(d+c-b-a)} \\ \left(\frac{2}{3}\left(c^3 - b^3\right)\right) + \frac{(d-c)}{(d+c-b-a)}\left(\frac{1}{3}c^2 + \frac{1}{6}(c+d)^2\right) \\ -(\mathrm{EVtrapezoidal})^2 \end{array} \right)^{0.5} \tag{7}$$

And its standard deviation can also be used to measure the variation of the fuzzy output from the expected value. The standard deviation for trapezoidal fuzzy number (a,b,c,d ) can be calculated using Equation (8):

$$\sigma(a,b,c,d) = \frac{2(d-a)+c-b}{4} \tag{8}$$

It is important to note that the measures described in Equations (5) to (8) can be used in assessing the uncertainty associated with project cost, in terms of its imprecision and vagueness. Tables 2 to 4 provide an illustrative example of comparative results generated under different scenarios.

## 4. APPLICATION OF THE DEVELOPED METHOD TO NON-REPETITIVE PROJECTS

This example project was drawn from the literature (Ahuja, 1984) and was analyzed to demonstrate the use of the developed method and to illustrate its essential features. The project schedule consists of 13 activities connected through eight events, as shown in Fig. 5. The schedule compression data is shown in Table 1. The project has a normal duration of 70 days and a direct normal cost of $6600. Indirect cost is estimated to be $1000 over the first 60 days and runs at a rate of $100 per day till the end of the project duration.

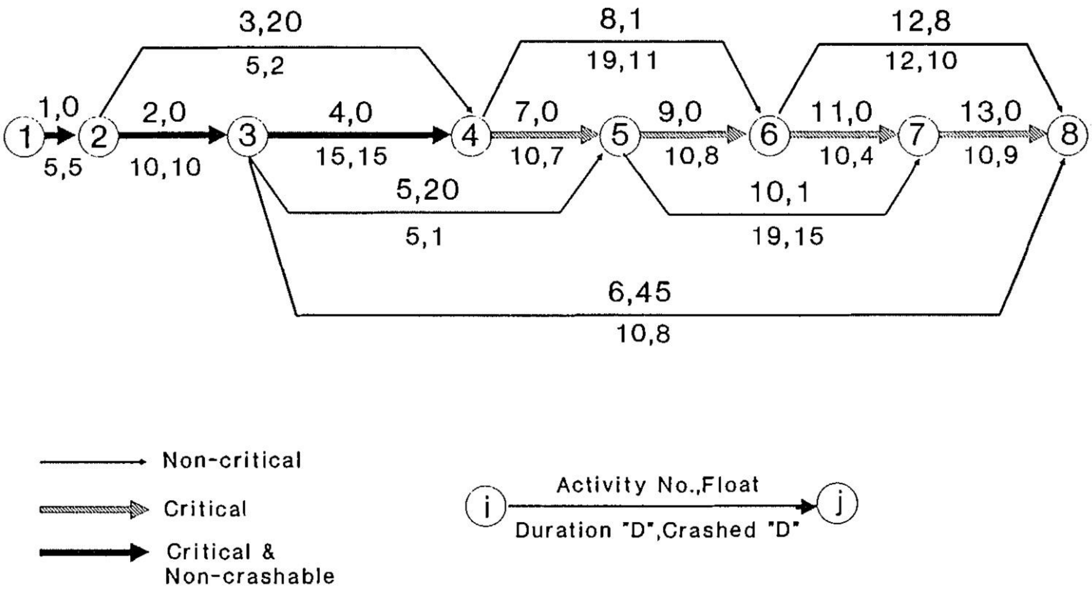


*FIG. 5: Example project CPM network*

The developed method was applied to analyze three possible scenarios for schedule compression:

[1] The first scenario is the base case scenario in which the target is to identify the least cost schedule, while the cost and the contractor's judgement are both considered but uncertainties associated with the crash cost are neglected. The cost is set to be more important than contractor's experience and their importance is set to 0.3 and 0.7, respectively.

[2] The second scenario is identical to the first scenario, except for the consideration of the uncertainties associated with crash costs of critical activities and for performing risk analysis on the selected compression plan.

[3] The third scenario is set to determine the least cost compression plan that meets targeted project duration of 67 days, under the same conditions of scenario 2.

*TABLE 1:* Schedule compression data

| Act. no | Act. status | Normal duration (d) | Crash duration (d) | Normal cost $ | Crash cost $ | CS | Trapezoidal representation of crash cost | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | a | b | c | d |
| 1 | Non Crash | 5 | 5 | 150 | 0 | ---- | - | - | - | - |
| 2 | Non-Crash | 10 | 10 | 200 | 0 | ---- | - | - | - | - |
| 3 | | 5 | 2 | 250 | 60 | 20 | - | - | - | - |
| 4 | Non-Crash | 15 | 15 | 900 | 0 | ---- | - | - | - | - |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| 5 | | 5 | 1 | 750 | 400 | 100 | 350 | 400 | 450 | 500 |
| 6 | | 10 | 8 | 1000 | 250 | 125 | 240 | 250 | 250 | 260 |
| 7 | Crushable | 10 | 7 | 300 | 240 | 80 | 230 | 235 | 240 | 260 |
| 8 | | 19 | 11 | 400 | 560 | 70 | 550 | 580 | 580 | 600 |
| 9 | Crushable | 10 | 8 | 500 | 100 | 50 | 90 | 110 | 130 | 150 |
| 10 | | 19 | 15 | 600 | 300 | 75 | 280 | 300 | 300 | 320 |
| 11* | Crushable | 10 | 4 | 700 | 510 | 85 | 500 | 520 | 520 | 540 |
| 12 | | 12 | 10 | 600 | 200 | 100 | 180 | 200 | 220 | 240 |
| 13* | Crushable | 10 | 9 | 250 | 50 | 50 | 40 | 50 | 70 | 80 |
| Total | | | | 6600 | 2670 | | | | | |

**Outputs**

***Scenario 1***

In this scenario, activity crash costs are taken as crisp numbers (see Table 1, column 6). The cost slope for each of the activities considered for compression is first calculated, and then activity crashing priorities are assigned (1 to 5) based on their respective cost slopes as described earlier. The contractor then assigns, based on his judgment, a priority value of 5 to activity 13 as favorable activity to be compressed first although it does not have the lowest cost slope, and he also assigns relative weights, as stated earlier, to these two priorities (i.e., cost and experience) to reflect the relative importance of each. In this example, applying Equation (2), the overall priority of activity (13) = 4.7, therefore, this activity is to be compressed first. This process continues until the least cost strategy plan is found. The method predicted the optimum project duration at 68 days and the project minimum total cost at $ 8500.

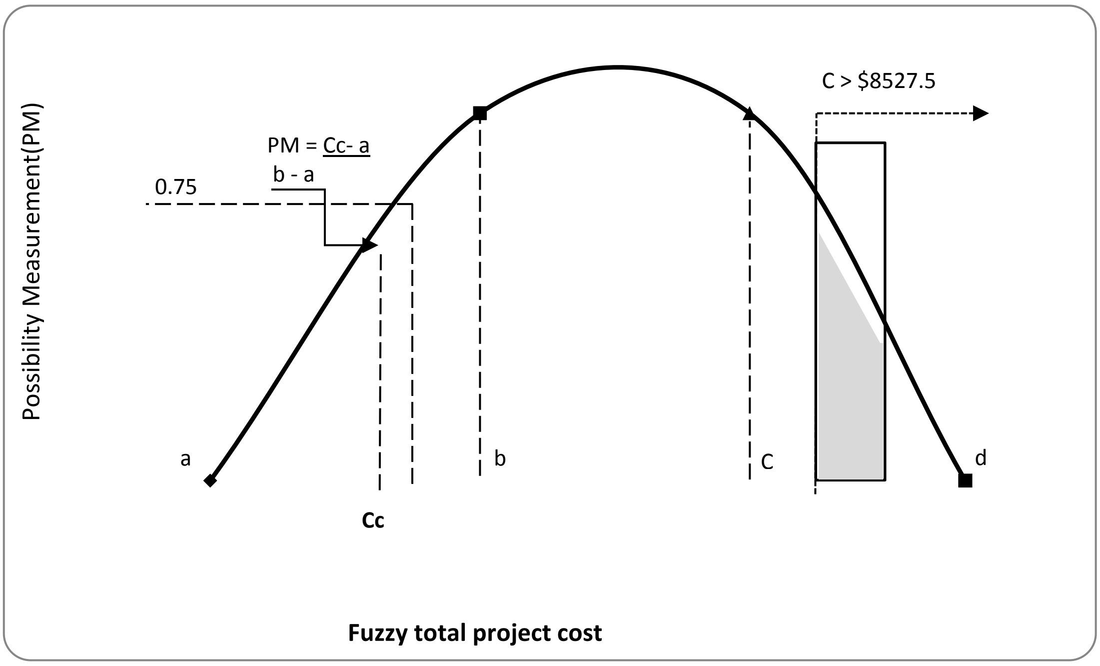


*FIG. 6: Fuzzy cost for least cost acceleration plan (scenario 2)*

***Scenario 2***

In this scenario, the method predicted the optimum project duration to be 68 days and the expected value of the project minimum total cost to be $8519.43. The fuzzy total cost is {8505,8515,8525,8533} dollars (Fig. 6). The most possible and plausible total project cost of that compression plan, as expressed by the fuzzy number shown in Fig. 6, is somewhere between $8515 and $8525. The possibility measure is applied to evaluate the possibility of having a targeted crisp value for that schedule compression plan. For example, the possibility of the project total cost of the selected compression plan being $8512.50 is equals to 0.75. The following two possibilities were also examined:

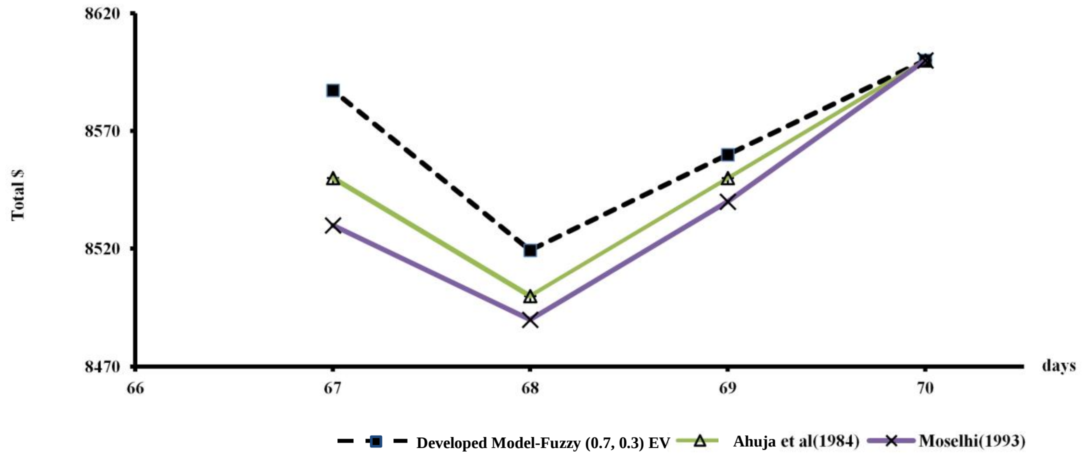


*FIG. 7*: Comparison of the results

- What is the possibility of the cost of the generated schedule compression plan falling between $8527.50 and $8530?
- What is the possibility of the cost of the generated schedule compression plan being exactly $8527.50?

In addressing the first question, the project cost is considered against the first event, which is expressed as {8527.5, 8527.5, 8530, 8530} and then against the project compression cost in the second event; expressed as {8527.5, 8527.5, 8527.5, 8527.5}. As for the first case, the elements which are included in the intersection range along with their associated degrees of memberships are: {8527.5|0.68, 8527.5|0.68, 8530|0.37, 8530|0.37}. In this case, the possibility that the project compression cost falls between 8527.5 and 8530 dollars is 0.68. Similarly, the possibility of having the cost of the compression plan exactly $ 8527.5 is 0.68 for the intersection of the two membership functions in this case is {8527.5 |0.68, 8527.5 |0.68, 8527.5 |0.68, 8527.5 |0.68}. The possibility measure pertinent to the two events takes its value from the maximum membership function value resulting from the intersection area of the two events involved. It is important to note that the possibility measure does not consider the size of their intersection area, which may lead to high value of a possibility measure while the intersection area is small. It is interesting here to observe that while the possibility measure in the two cases analyzed above is identical (i.e., PM=0.68), their respective areas of intersection are not. The Agreement Index, on the other hand, accounts for that area of intersection and is designed to provide complimentary information to the possibility measure. Tables 2 and 3 depict a comparison between the outputs of the developed method and those generated by Ahuja et al. (1984) and Moselhi (1993). It can be seen from Table 3 that the developed method, unlike the other two methods used in the comparison, provides more useful information to the decision maker about the generated compression plan. The developed method offers contractors effective tools to evaluate the possibilities of not exceeding targeted cost of a given schedule compression plan and of having a plan that meets a defined targeted cost. The latter is not possible to determine using any heuristic or probability-based method. The generated possibility measure, agreement index, expected value, fuzziness measure, variances and ambiguity measure provide useful information to describe the uncertainty associated with the cost of the generated compression plan.

*TABLE 2:* Comparison of the Results of Scenario 2

| Method | Input distribution | Output distribution | Total cost | EV |
|---|---|---|---|---|
| Ahuja et al. (1984) | Crisp | Crisp | 8500 | 8500 |
| Moselhi (1993) | Crisp | Crisp | 8490 | 8490 |
| Developed Method | Trapezoidal | Trapezoidal | (8505,8515,8525,8533) | 8519.43 |

*TABLE 3:* Evaluation of Different Measures applied to Scenario 2

| Method | Possibility C >8527.5 | Possibility C=8525 | Is possibility of C=8530 is > that C=8525? | F(A) | AG | Vari(Trape) | σ |
|---|---|---|---|---|---|---|---|
| Ahuja et al. (1984) | NA | NA | NA | 0 | NA | NA | 0 |
| Moselhi (1993) | NA | NA | NA | 0 | NA | NA | 0 |
| Developed Method | 0.098 [e] | 1.0 [f] | No [f] | 9 | 8 | 6.08 | 16.5 |

[e] agreement index [f] possibility measure Vari(Trape): variances measures C = total project cost of compression plan

**Scenario 3**

In this scenario, the developed method is used to predict the cost of compressing project schedule to a targeted duration of 67 days. The predicted project fuzzy total cost is presented in Fig. 8. The method predicts the expected total project cost of the targeted schedule compression to be $8587.33. Applying the possibility measure reveals a 0.82 possibility that the cost of project schedule compression will be at $8580 and that the most possible and plausible compression project cost is somewhere between $8582.33 and $8592.33. The results obtained using the developed method are summarized in Table 4.

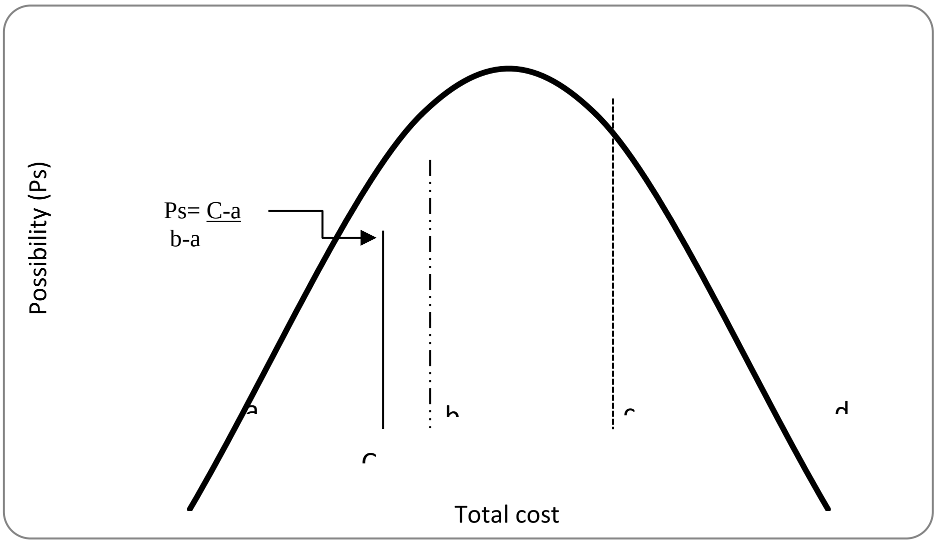


*FIG. 8: Fuzzy cost of the compression plan (target= 67 days)*

*TABLE 4:* Summary of the method outputs

| Scenario | Plan Cost | Uncertainty $ | EV | F(A)) | AG | Vari(trape) | σ |
|---|---|---|---|---|---|---|---|
| 1 | 8500.00 | NA | 8500.00 | 0 | 0 | NA | 0 |
| 3 | 8505,8515, 8525,8533 | Yes | 8519.43 | 9 | 8 | 6.08 | 16.5 |
| 4 | 8569,8582.33, 8592.33,8605.67 | Yes | 8587.33 | 13.33 | 11.11 | 7.76 | 20.83 |

## 5. SUMMARY AND CONCLUDING REMARKS

This paper presents a new method developed for schedule compression of non-repetitive construction projects. The method accounts for contractors' judgment in addition to cost slope by generating combined priorities for activities to be compressed. The method accounts for uncertainties associated with crash cost of critical activities using fuzzy set theory (FST). A construction project network schedule drawn from literature was analyzed to demonstrate the use of the developed method and to illustrate its capabilities. The results prove that (1) the developed method can produce more practical compression plans that meet the level of possibility measures set by the user, and (2) FST based methods can be used effectively for schedule compression; accounting for uncertainties in a much easier and faster way than probabilistic based models and addressing scenarios probabilistic methods cannot address. This is due to the fact that unlike probabilistic methods the developed method doesn't require data gathering, also it has the ability to respond to the possibility of an event occurring at any specified date. The method is capable of generating direct solutions and of facilitating the use of experience and knowledge of members of project teams by requesting them to define suitable fuzzy membership functions to the operations at hand.

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