INTEGRATING FUZZY-LOGIC DECISION SUPPORT WITH A BRIDGE INFORMATION MANAGEMENT SYSTEM (BRIMS) AT THE CONCEPTUAL STAGE OF BRIDGE DESIGN

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SUMMARY: In recent years, infrastructure restoration has been backlogged with complex factors that have captured the attention of municipal and federal authorities in North America and Europe. The subjective nature of evaluating bridge conditions and bridge deterioration is one of the main factors that influences bridge maintenance, repair, and replacement (MR&R) decisions. This study presents a stochastic fuzzy logic decision support integrated with a bridge information management system (BrIMS) to forecast bridge deteriorations and prioritize maintenance, repair, and replacement (MR&R) decisions at the conceptual design stage. The proposed system considers numerous factors that influence the prioritization of bridge MR&R decision making including complex time-dependent gamma shock models. A parametric analysis is conducted in order to quantify the degree of accuracy of the system. Implementation of the system platform demonstrated the viability of integrating BrIMS with fuzzy-logic deterioration forecast techniques at the conceptual stage of bridge design. The proposed system is validated through a case study and found to be in agreement with actual bridge deterioration results with a percentage difference of approximately 10 - 15 %. Besides that, the integrated platform may be utilized as a forecasting tool that is capable of predicting and prioritizing MR&R decisions to components for diverse bridge design alternatives.

KEYWORDS: Fuzzy-logic, Decision Support, Deterioration Forecast, Bridge Information Model, Bridge Information Management System


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1. INTRODUCTION

Typically, forecasting bridge infrastructure deterioration from distinct condition assessments and statistics is a challenging task. Due to the highly complex and erratic nature of infrastructure data, deterministic bridge deterioration models are quite often not applicable. Temporal reliability analysis “hazard functions,” such as Markov chains, Bayesian networks, and gamma models, have been developed for bridge and storm sewer systems. Predicting bridge deterioration conditions is the main constituent of infrastructure asset management techniques. Furthermore, a decision support system based on fuzzy-logic theory that assists asset managers in making appropriate MR&R decisions is vital (Wang et al., 2015). Bridge performance indicators should be based on bridge beneficiaries’ perceptions of technical parameters. Integrating these indicators with stochastic time-dependent modeling of bridge deteriorations is important for planning and prioritizing MR&R activities. These activities may include inspection, sampling, preventative and maintenance operations. Based on the aforementioned, a time-dependent prediction of the overall bridge deteriorations necessitates the development of a thorough, reliable, and user-friendly fuzzy-logic decision support system.

1.1 Problem Definition

Nowadays, most bridge information management methodologies are strictly based on life cycle analyses offset by available funds and budget constraints. Repair costs, in many situations, have proven to exceed annual or semi-annual preventive maintenance costs (Liang et al., 2002). Most bridge stakeholders are reluctant to pay for preventive maintenance, which appears to be of no benefit or which bridge asset managers have found from experience to be unsuccessful in preventing a bridge structure from deteriorating.

In an attempt to overcome this shortcoming, this study is intended to demonstrate the viability of stretching bridge information models to capture the conceptual design of a bridge while applying sensitivity analyses to identify the most sensitive elements and subsequently to forecast bridge elemental deteriorations. Furthermore, it is assumed that integrating a fuzzy logic decision support system with BrIMS and deterioration forecast for bridges is possible only if its objectives are kept simple, focused, and organized. Therefore, basic straightforward bridge information management (BrIMS) processes have been researched, recalled, and analyzed. According to Bentley bridge solutions, the eight processes of BrIMS are: (1) bridge type selection; (2) 3D CAD model; (3) technical analysis; (4) planning for construction; (5) production; (6) phases of construction; (7) maintenance; and (8) remediation (Peters, 2009). Extending research published by Wang and Elhag (2008) and Cheng and Hoang (2012) that proposed the integration of BrIMS with fuzzy systems to prioritize MR&R solutions for deteriorating bridges, authors of this study introduce the idea of a fuzzy-logic decision support framework integrated with gamma shock modelling for forecasting bridge deteriorations. The main advantage of such integration underlies the benefit of capturing economical preventative maintenance routes and making strategic MR&R decisions at the conceptual design stage.

Whilst several attempts have been made at modeling the deterioration of bridge infrastructures, practical developments were recorded recently, where researchers employed the gamma stochastic process effectively to temporal deteriorations and subsequently implemented into MR&R decision support systems. Furthermore, earlier research work had not considered the integration of decision support systems with gamma shock models at the conceptual design stage. In contrast, researchers had focused more on improvement factors for enhancing bridge maintenance and information management techniques.

In this study, the following processes; i) bridge type selection; ii) 3D CAD model; iii) technical analysis; and iv) maintenance and remediation are selected for the development and integration of gamma shock models BrIMS at the conceptual design stage. The proposed system integrates quality functions for maintenance, repair, and replacement (MR&R) alternatives with a Gaussian probabilistic matrix factorization. The resulting system produces competitive priority ratings that eliminate ambiguities in bridge life cycle evaluation. Hence, the proposed integrated information management system becomes a new approach to informing downstream processes of bridge projects at the conceptual design stage.
1.2 Research Objectives
The integrated approach presented herein may be utilized to plan the maintenance and to monitor the deterioration of bridges and then to prioritize the maintenance, repair, and replacement alternatives; (i.e., inspection, sampling, preventative, and maintenance operations). This integration technique is a rational approach that justifies most “ineffective” spending by bridge stakeholders, since it considerably reduces subjectivity in quantifying bridge deterioration. Moreover, the integration not only contributes to the reliability of a particular bridge element but also to the reliability of the collected data and the probability of occurrence of deterioration benchmarks such as corrosion and elemental degradations.

The main objective of this study, then, is to develop fuzzy logic decision support system using complex quality functions and a gamma stochastic deterioration model that is based on the integration of probabilistic models in an attempt to improve the effectiveness of bridge information management systems. Towards that goal, a wider insight into the integration of a decision support system can assist bridge asset managers in proposing a strategic solution to deteriorating bridge infrastructures.

1.3 Literature Review
Highway and transportation authorities have often relied upon deterministic deterioration curves for predicting bridge maintenance programs. In past years, bridge infrastructure management systems have been modeled using traditional Markov chain models. In the past three decades, inconsistent MR&R decisions for bridges and road infrastructures have necessitated the evolution of further investigative studies in the modeling of bridge deteriorations in parallel with developing reliable decision support solutions (Lounis, 2000).

According to earlier studies conducted by Golabi et al. (1993) and Hawk (1999), incremental degradation of bridge components were designed by a static Markov chain model process in which accumulating deficiencies following a stress cycle were assumed to depend on original conditions and duration of the cyclic stress loading only. However, a study conducted by Madanat et al. (1997) confirmed that most bridge deterioration models were not static and proposed descriptive variables that traditional Markov models must take into account to develop more realistic models. Furthermore, Lounis (2000) restated the capability of Markov models in predicting the remaining service life of a bridge at any time based on existing deterioration conditions.

Recent bridge information management systems have implemented Markov chains deterioration models which is considered a major step forward towards incorporating stochastic-natured deterioration models. Moreover, several bridge maintenance aspects and corresponding bridge asset visual inspection standards and procedures endorsed the implementation of the Markov chain model which is known to possess restrictive assumptions. Although traditional Markov chain models are proven practical and relatively simple to develop, they do possess limitations especially at comprehensive project phases and are considered not sufficient for analyzing critical structures when it comes to safety matters. The most important limitation, however, is the deployment of elemental condition crisp rating systems based on vague performance indicators mainly influenced by the scale of subjectivity involved in the visual inspection and not explicitly related to qualitative and quantitative parameters such as material properties, stress-strain conditions, and structural behavior. Proceedings and evolutions that tailored the Markov chain approach in bridge information management systems were provided in a study published by Frangopol et al. (2001). In a later study later, Lounis and Madanat (2002) presented a two-level decision support system that amalgamates stochastic deterioration models to enhance efficacy of bridge maintenance management systems. The first level management is based on Markov models that pin points perilously damaged structures and predicts short- and long-term deterioration and essential maintenance at a bridge-level and network-level. While the second level management is based on mechanistic models that target considerably deteriorated bridge structures classified from the first level management and assess their integrity, serviceability, and maintenance.

In an attempt to overcome Markov chain model limitations, reliability-based deterioration models that are based on gamma shock models have been researched and recalled. One study conducted by Pandey and Van Noortwijk (2004) investigated a gamma model for temporal structural reliability and presented a relative assessment of random variable deterioration models based on the first order reliability methods and temporal stochastic modeling based on the gamma process. In their paper, the authors employed a stochastic model to count for both sampling and time-dependent variances allied with a structural system deterioration process. A detailed comparison of lifetime probability and cumulative density functions as well as survival curves between random variable and

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gamma process mechanisms were presented. It was concluded that the random variable model overestimates probability of remaining life time of a particular structure in the long-run and gamma model provides more reasonable estimates of life times which shall enhance the implementation of such stochastic deterioration models more often in structure reliability analysis. In another study, Van Noortwijk et al. (2005) examined a gamma process model for temporal structural reliability and presented a combined computational method comprising both deterioration resistance and variable load modeled as a stochastic gamma process. It is concluded that the time at which the deteriorating resistance falls below the fluctuation load cumulative distribution function can be formulated as a functional equation which could be solved numerically by applying a series of integration and partial derivations to simulate deterioration paths of the generalized gamma process. It was also found that the proposed method contributes to the ‘well-fit’ of structural monotonic aging peaks-over-threshold distributions with extreme value figures. Furthermore, Van Noortwijk et al. (2007) examined gamma processes and peaks-over-threshold distributions for time-dependent reliability. In their paper, a comprehensive discussion on the evaluation of structural reliability was presented where a methodology that integrated two stochastic processes originating from a Poisson process for obtaining the temporal reliability of a particular structural component was proposed. Another study conducted by Edirisinghe et al. (2013) presented the application of gamma process for stochastic deterioration prediction of building elements derived from discrete condition data obtained from the Victorian local government infrastructure asset database. The focus of their study was geared to develop a complex and more reliable deterioration prediction system for managing their building assets. Gamma process probability and cumulative density functions were derived and plotted in addition to building elements predicted temporal deterioration. At the end, the authors concluded with the capability of the proposed gamma process deterioration model for forecasting deterioration of building elements with time by incorporating building condition and deterioration highly scattered data. Moreover, Reddy and Ramudu (2013) analyzed a numerical arithmetic-geometric maintenance model for deteriorating system subject to a random environment. Their main goal was to develop a replacement model for a particular deteriorating system in a random environment while utilizing an arithmetic-geometric approach that maximizes long-run anticipated payoff within a cycle time. System replacement average cost rate versus the replacement policy were obtained and plotted where the peak of the curves explicitly indicates an optimal replacement policy. At the end, the authors concluded that by varying the parameters of the developed model, the optimal number of failure only impacts the long-run anticipated payoff cost per cycle time.

In summary, there have been substantial efforts to apply a gamma model to the perseverance of bridge structures. Although advancements in deterioration modeling have influenced bridge MR&R solutions, arguments for its application to time-dependent incremental degradations of bridge elements still exist. This study presents the proposal of a reliability-based deterioration model based on the deployment of complex quality functions originating from the house of quality (HOQ) model. The proposed decision support system is based on quantitative and qualitative fuzzy logic scorings that take into account technical, functional, and safety parameters. Moreover, such systems are powerful in the manner that they are capable of analyzing single or multiple complicated bridge elements of a highway bridge that possess diverse failure modes.

2. SYSTEM ARCHITECTURE

The proposed methodology comprises an innovative bridge information management system (BrIMS) based on a framework that is capable of integrating bridge gamma stochastic deterioration modeling with a fuzzy-logic decision support system. The framework is developed by deploying complex quality functions derived from bridge beneficiary-driven parameters and symmetrical triangular fuzzy numbers (STFN’s) to capitulate bridge evaluation ambiguities. Furthermore, the proposed system possesses a unique aspect of BrIMS by incorporating diverse bridge MR&R solutions into a multi-criteria decision making approach (MCDM) to derive competitive priority ratings. A schematic view of the interrelations among the 3D computer-aided design (CAD) solutions with the developed bridge deterioration forecast system is illustrated in Fig. 1.
As illustrated in Fig. 1, it is important to note that the proposed integrated system is part of an integrated preliminary fuzzy-logic decision support system developed earlier by the authors of this study. The following two modules:
- Module 1 – Conceptual Bridge Design
- Module 4 – Deterioration Forecast

are an integral part of this study whereas the highlighted items that correspond to the following three modules:
- Module 2 – Fleet Selection
- Module 3 – Preliminary Cost Estimation
- Module 5 – Bridge Line of Balance

are not part of this study. The proposed system is developed in an object-oriented .NET framework and undertaken by completing the following six main steps:
1) Data collection of bridge-user-driven parameters.
2) Implementation of a decision support system that assists the user in making MR&R decisions.
3) Development of complex quality functions to evaluate bridge users’ relative perception of bridge components.
4) Deployment of a numerical model to evaluate bridge MR&R ratings.
5) Development of a mean deterioration resistance regression fit where MR&R rankings are determined.
6) Optimizing and prioritizing maintenance, repair, and replacement alternatives.

At first, the user inputs importance ratings on bridge components. Following bridge user’s assessment on the importance of bridge components on bridge design alternatives, importance perception ratings are determined. Afterwards, a bridge users’ competitive matrix is developed, where the probability distribution and corresponding measure of entropy of bridge components is determined. Once completed, the user proceeds with inputting a set of improvement goals that represent the user’s required improvement in performance of bridge components. Following the input of goals, QFD and TOPSIS analyses are undertaken in order to develop priority rankings of bridge components. Afterwards, the user is guided to the HOWs scoring input form, which represents bridge users’ importance ratings on bridge maintenance, repair, and replacement alternatives (MR&R) based on the output of...
QFD and TOPSIS analyses on bridge components. Following bridge user’s assessment on the importance of bridge components on MR&R alternatives, importance perception ratings are obtained. Afterwards, a bridge users’ competitive matrix is developed, where the probability distribution and corresponding measure of entropy along with competitive ranking of bridge components is determined. Afterwards, the user inputs the year digit and corresponding mean deterioration percentages such that a regression analysis along with the quality of fit methodology is deployed. Once completed, the developed system provide the user a recommendation statement to reconsider the performance of a bridge component in order to enhance its deterioration resistance capacity, which represents a component’s remaining service life. Fig. 2 summarizes the deterioration forecast module process flowchart.

FIG. 2: Deterioration Forecast Module Process Flowchart

As illustrated in Fig. 2, the developed module is implemented in an object-oriented .NET framework and undertaken by completing the following five main steps:
1) Data collection of bridge type and geometric selection imported from Module 1.
2) Implementation of a decision support system that assists the user in making MR&R decisions.
3) Development of complex quality functions to evaluate bridge users’ relative perception of bridge components.
4) Deployment of a numerical model to evaluate bridge MR&R ratings.
5) Prioritizing maintenance, repair, and replacement (MR&R) alternatives.

The flow of geometric information for diverse bridge types and resistance deterioration predictions begins at fuzzy logic scorings and ends at the forecasting of bridge component deterioration based on its cost recovery period. Throughout the process, the deployment of the technique of preference by similarity to ideal solution (TOPSIS), a multi-criteria analytical approach utilized for the selection of the MR&R alternative based on a specified list of parameters is undertaken. The MR&R are identified based on performance condition assessments of bridges in operational stages and grouped into three main categories as follows; Category (I) is a ‘Maintenance’ category that includes the maintenance of a bridge component for an expected extent ranging between 15% to 45% and comprising the decisions; ‘Maintenance: S1.M15’, ‘Maintenance: S2.M30’, and ‘Maintenance: S3.M45’; Category (II) is a ‘Repair’ category that includes the repair of a bridge component for an expected extent of deterioration ranging between 45% to 75% and comprising the decision; ‘Repair: S4.REPA45’, ‘Repair: S5.REPA60’, and ‘Repair: S6.REPA75’; Category (III) is a ‘Replacement’ category that includes the replacement of a bridge component for an expected extent of deterioration ranging between 75% to 100% and comprising the decisions; ‘Replacement: S7.REPL75’, ‘Replacement: S8.REPL90’, and ‘Replacement: S9.REPL100’. The MR&R alternatives are defined as cost representatives of a bridge component. For instance, a maintenance ‘M15’ alternative represents ‘15%’ of a bridge component’s estimated cost.
2.1 Fuzzy Logic Decision Support System

Due to the scarcity of bridge deterioration data, it is necessary to develop a fuzzy logic scoring system in order to assist bridge stakeholders and designers in predicting bridge deterioration at conceptual design stages. Otayek et al. (2012) have studied the integration of a decision support system based on a proposed machine technique as part of artificial intelligence and neural networks (NN). In their study, the authors recommend continuous and further development in decision support systems in an attempt to assist bridge designers in predicting bridge deteriorations at conceptual phases. On the other hand, Malekly et al. (2010) have proposed a methodology of implementing a quality function deployment (QFD) technique and a technique of preference by similarity to ideal solution (TOPSIS). Their methodology is integrated in a novel oriented approach while overcoming interoperability issues among the disperse databases. Furthermore, Tee et al. (1988) studied the viability of developing a numerical approach based on fuzzy set rules such that the degree of subjectivity involved in evaluating bridge deterioration was treated systematically and was incorporated into a systematic knowledge-based system. Liang et al. (2002) proposed grey and regression models for predicting the remaining service life of existing reinforced concrete bridges. In their study, the fuzzy logic concept was introduced as a methodology for evaluating the extent of deterioration of existing bridge structures. Zhao and Chen (2002) proposed a fuzzy logic system for bridge designers to help to predict bridge deteriorations based on factors incorporated at the initial design phase. Sasmal et al. (2006) recalled earlier studies using fuzzy logic theory and stated that those methodologies were either much too simple or too complex so that key support requires considerable time. These studies; however, overlooked key issues pertaining to membership functions and other parameters, such as priority vectors and mappings, which are fundamental for bridge condition assessments. Therefore, the authors of the present study propose an integrated system for deterioration evaluations of bridges, based on fuzzy mathematics integrated with an eigen-vector technique and priority ratings. In this study, the proposed system is anticipated to be of novelty to BrIMS integrated technologies and possess a great advantage over the diverse deterioration forecast algorithms, prototypes, and systems presently used in the bridge construction industry by including a fuzzy logic decision support system based on quality functions deployment (QFD) and the technique of order preference by similarity to ideal solution (TOPSIS). As a result, competitive priority ratings of bridge components alternatives are produced rather than completely including or excluding alternative solutions at the conceptual design stage. Fig. 3 illustrates a high-level process of the fuzzy logic decision support system integrated with the BrIMS.

As shown in Fig. 3, the proposed system includes a fuzzy logic decision support that extracts information from the 3D BrIM tool via a DLL-invoked API method that automatically recalls the parametric enriched object-oriented model. For instance, the system provides the user with an option to develop an information module by utilizing the fuzzy logic scoring system in order to determine the bridge type based on the deployment of the QFD and TOPSIS processes; otherwise, the application automatically extracts data from the BrIM model and presents nominations and recommendations of selected bridge type based on technical and functional spans and geotechnical attributes. Furthermore, the system is hard-coded to extract all necessary information from the assigned model by exporting BrIM input databases via the Industry Foundation Classes (IFC) file format, which reduces loss of information during file transmission. After that, the system is objectively developed for bridges such that capturing of data displayed in the calling software is conducted by utilizing BrIM objects. Finally, bridge element attributes are recalled and organized via a DLL-invoked programming language and incorporated into the SQLite database server.
2.1.1 Quality Functions

Conceptual bridge design is found to be significantly influenced by each of the following nine main components: (1) approach slab ‘C1’; (2) deck slab ‘C2’; (3) expansion joint ‘C3’; (4) parapet ‘C4’; (5) girder ‘C5’; (6) bearings ‘C6’; (7) abutment ‘C7’; (8) pier ‘C8’; and (9) foundation ‘C9’. Selection of the components is based on critical factors that bridge designers rely upon and bridge users’ perception on the importance of components. Hence, a 9-point symmetrical triangular fuzzy logic numbers (STFN) ranging from one to nine, with one being very low and nine being very high, is adopted for assisting the decision maker in predicting bridge users perception pursuant to the main nine bridge components listed above. The scoring system comprises crisp and fuzzy measures when uncertainty arises.

Where for instance, [0,2] indicates the range of fuzziness of the crisp score ‘1’. Similarly, [8,10] represents the range of fuzziness of the crisp score ‘9’. Afterwards, bridge users are identified and categorized as follows: (i) stakeholders/government; (ii) designers/engineers; (iii) contractors/builders; and (iv) public/residents. Also, the following nine common bridge types ‘alternatives’ are identified and incorporated into the database platform for QFD analyses: (1) beam bridges ‘W1’; (2) truss bridges ‘W2’; (3) cable-stayed bridges ‘W3’; (4) tied-arch bridges ‘W4’; (5) arch bridges ‘W5’; (6) suspension bridges ‘W6’; (7) double-decked bridges ‘W7’; (8) movable bridges ‘W8’; and (9) cantilever bridges ‘W9’. The adopted QFD analytical technology utilized for the selection of bridge type is presented in Fig. 4.
Upon completion of user scorings on the nine bridge components, perception on relative importance ratings of the components is determined. In this study, Chan and Wu (2005) numerical methodology is deployed due to its efficiency, systematic characteristics, and ease of use in competitive analysis of bridge components selection. Crisp and measure forms of expected relative importance ratings are obtained in accordance with Chan and Wu (2005) equations (1) and (2):

\[
g_{mk} = \frac{g_{m1} + g_{m2} + g_{m3} + g_{m4} + g_{m5} + g_{m6} + g_{m7} + g_{m8} + g_{m9}}{9} \tag{1}
\]

\[
\tilde{g}_{mk} = \frac{\tilde{g}_{m1} + \tilde{g}_{m2} + \tilde{g}_{m3} + \tilde{g}_{m4} + \tilde{g}_{m5} + \tilde{g}_{m6} + \tilde{g}_{m7} + \tilde{g}_{m8} + \tilde{g}_{m9}}{9} \tag{2}
\]

Where; \( g_{mk} \) is a bridge user relative importance perception on a component in ‘crisp’ form, \( k \) is a bridge user, \( \tilde{g}_{mk} \) is a bridge user relative importance perception on a component in ‘fuzzy’ form. In other words, \( g_{mk} \) is the average “integer” crisp scoring value of a bridge user on the relative importance of each of the components and \( \tilde{g}_{mk} \) is the average “integer” fuzzy scoring value of a bridge user on the relative importance of each of the components. Following the determination of relative importance ratings, bridge users competitive comparison matrix analysis is developed as per Chan and Wu (2005) equations (3) and (4):

\[
X = \begin{bmatrix} x_{mk} \end{bmatrix} \in \mathbb{R}^{9 \times 9} \tag{3}
\]

\[
x_{mlk} = \frac{x_{m11} + x_{m12} + x_{m13} + x_{m14}}{4} \tag{4}
\]

Where; \( X \) is the bridge users comparison matrix, \( x_{mk} \) is a bridge user assessment on \( C_m \), \( x_{mlk} \) is a bridge user assessment of a bridge alternative on \( C_m \), and \( C_m \) is a bridge component. Afterwards, the probability distribution of each \( C_m \) on bridge alternatives is calculated using Chan and Wu (2005) equation (5):
\[ p_{mk} = \frac{x_{mk}}{x_m} \]  

(5)

Where; \( p_{mk} \) is the probability distribution of \( C_m \) on bridge alternatives, \( x_{mk} \) is a bridge user assessment on \( C_m \) ‘result obtained from equation (4)’, and \( x_m \) is the total of bridge users assessment of all bridge alternatives on each of \( C_m \). Following the determination of probability distribution of \( C_m \), its measure of entropy, which is a quantification of the expected value of a system with uncertainty in random variables, may be obtained using Chan and Wu (2005) equations (6) and (7):

\[
E(C_m) = -\phi_b \sum_{m=1}^{9} p_{mk} \ln(p_{mk})
\]  

(6)

\[
\phi_b = \frac{1}{\ln(9)}
\]  

(7)

Where; \( E(C_m) \) is the measure of entropy by a discrete probability distribution for \( C_m \), \( \phi_b \) is the normalization factor that guarantees \( 0 \leq E(p_1, p_2, \ldots, p_L) \leq 1 \). \( p_{mk} \) is the probability distribution of \( C_m \) for the diverse bridge alternatives. Higher entropy or \( (p_1, p_2, \ldots, p_L) \) implies smaller variances and lesser information in a probability distribution \( p_L \). At the end, bridge alternatives’ weights on each of the nine \( C_m \) is calculated based on Chan and Wu (2005) equation (8):

\[
e_m = \frac{E(C_m)}{\sum_{m=1}^{9} E(C_m)}
\]  

(8)

Where; \( e_m \) is the importance weight of bridge component, \( C_m \), and \( E(C_m) \) is the measure of entropy by a discrete probability distribution for \( C_m \). This complex quality function deployment mechanism of assigning priorities to competing alternatives is directly related to information theory concept of entropy. Once completed, a set of improving goals strategy on each of the bridge components to enhance the bridge alternative deterioration resistance performance is defined. The performance goals on the bridge components are identified based on the 9-point STFN scale as per Chan and Wu (2005) equation (9):

\[
i = (i_1, i_2, i_3, i_4, i_5, i_6, i_7, i_8, i_9)
\]  

(9)

Where; \( i \) is the improvement goal set. It is important to note that the improvement goals must be higher than the initial performance rating of a bridge component, \( C_m \) for a bridge alternative, \( W_{mk} \). This implies that in case the initial rating of a component for a particular bridge alternative is high, the goal set must be higher to maintain its rating and enhance the competition amongst bridge alternatives. Otherwise, if the initial rating is lesser, then the improvement goal is set to improve the performance of the same and enhance its importance weight. Once improvement goals are set, an improvement ratio is calculated as per Chan and Wu (2005) equation (10):

\[
r_m = \frac{i_m}{x_{mk}}
\]  

(10)

Where; \( r_m \) is the improvement ratio, \( i_m \) is the improvement goal set, and \( x_{mk} \) is a bridge user assessment on \( C_m \) ‘result obtained from equation (4)’. The competitive rating for a bridge component, \( C_m \), in ‘crisp’ form is obtained as per Chan and Wu (2005) equation (11):

\[
f_m = i_m * g_m * e_m
\]  

(11)
Where; \( f_m \) is the competitive rating, \( \tilde{f}_m \) is the improvement goal set, \( g_m \) is a bridge user relative importance perception on a component in ‘crisp’ form, and \( e_m \) is the importance weight. The final importance rating for a bridge component, \( \tilde{f}_m \), in ‘fuzzy’ form is obtained as per Chan and Wu (2005) equation (12):

\[
\tilde{f}_m = \tilde{f}_m \cdot \tilde{g}_m \cdot e_m
\]

Where; \( \tilde{f}_m \) is the competitive rating in ‘fuzzy’ form, \( \tilde{f}_m \) is the improvement goal set, \( \tilde{g}_m \) is a bridge user relative importance perception on a component in ‘fuzzy’ form, and \( e_m \) is the importance weight. Once completed, technical measures to expected maintenance, repair, and replacement (MR&R) decisions to the deterioration of bridge components are grouped into three main categories as illustrated in Table 1.

### Table 1: Maintenance, Repair, and Replacement Decisions Versus Extent of Deterioration (%)

<table>
<thead>
<tr>
<th>Extent of Deterioration (%)</th>
<th>Category I</th>
<th>Category II</th>
<th>Category III</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>√</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>30</td>
<td>√</td>
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<td>45</td>
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<tr>
<td>60</td>
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<td>75</td>
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<tr>
<td>90</td>
<td>-</td>
<td>-</td>
<td>√</td>
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<tr>
<td>100</td>
<td>-</td>
<td>-</td>
<td>√</td>
</tr>
</tbody>
</table>

As illustrated in Table 1, category (I) is a ‘Maintenance’ category that comprises maintenance of bridge component for an expected extent ranging between 15% to 45% and comprising the decisions; ‘Maintenance: S1.M15’, ‘Maintenance: S2.M30’, and ‘Maintenance: S3.M45’; category (II) is a ‘Repair’ category that comprises repair of bridge component for an expected extent of deterioration ranging between 45% to 75% and comprising the decision; ‘Repair: S4.REPA45’, ‘Repair: S5.REPA60’, and ‘Repair: S6.REPA75’; category (III) is a ‘Replacement’ category that comprises replacement of bridge component for an expected extent of deterioration ranging between 75% to 100% and comprising the decisions; ‘Replacement: S7.REPL75’, ‘Replacement: S8.REPL90’, and ‘Replacement: S9.REPL100’. It is important to note that the proposed categories and extent of deterioration is for illustrative purposes and can be customized dependent upon the bridge location and the regional weather forecast. Similar to the determination of competitive comparison matrix analysis on bridge components, user comparison matrix analysis on technical measures for expected deterioration in ‘crisp’ and ‘fuzzy’ forms respectively are determined as per Chan and Wu (2005) equations (13) and (14):

\[
R = \left[ r_{mn} \right]_{10 \times 9}
\]

\[
\tilde{R} = \left[ \tilde{r}_{mn} \right]_{10 \times 9}
\]

Where; \( R \) is the comparison matrix on technical measures, \( r_{mn} \) is a bridge user technical measure assessment on \( C_m \) in ‘crisp’ form, \( \tilde{r}_{mn} \) is a bridge user technical measure assessment on \( C_m \) in ‘fuzzy’ form, and \( C_m \) is a bridge component. Hence, the technical rating for a measure, \( t_{mn} \), on a bridge component, \( C_m \) in ‘crisp’ form is obtained as per Chan and Wu (2005) equation (15):

\[
t_{mn} = \sum_{m=1}^{9} f_m \cdot r_{mn} \quad \text{where; } n = 1, 2, ..., 10
\]
Where; $f_{mn}$ is the technical rating on a measure in ‘crisp’ form, $f_{m}$ is the competitive rating ‘result obtained from equation 11’ in ‘crisp’ form, and $r_{m}^{\text{f}}$ is a bridge user technical measure assessment on $C_{m}$ in ‘crisp’ form. The technical rating for a measure, $r_{m}^{\text{f}}$, on a bridge component, $C_{m}$ in ‘fuzzy’ form is obtained as per Chan and Wu (2005) equation (16):

$$
\hat{r}_{mn} = \sum_{m=1}^{q} f_{m} \ast r_{mn}^{'}, \quad \text{where; } \ n = 1, 2, \ldots, 10
$$

Where; $\tilde{r}_{mn}$ is the technical rating on a measure in ‘fuzzy’ form, $\tilde{f}_{m}$ is the competitive rating ‘result obtained from equation 12’ in ‘fuzzy’ form, and $\tilde{r}_{mn}$ is a bridge user technical measure assessment on $C_{m}$ in ‘fuzzy’ form. Afterwards, the probability distribution of each $C_{m}$ on bridge deterioration technical measures is calculated using Chan and Wu (2005) equation (17):

$$p_{mn} = \frac{x_{mn}}{x_{m}}$$

Where; $p_{mn}$ is the probability distribution of $C_{m}$ on technical measure, $x_{mn}$ is a bridge user assessment on $C_{m}$ ‘result obtained from equation (4)’, and $x_{m}$ is the total of bridge users assessment of all technical measure on each of $C_{m}$. Following the determination of probability distribution of $C_{m}$, its measure of entropy, which is a quantification of the expected value of a system with uncertainty in random variables, may be obtained using Chan and Wu (2005) equations (18) and (19):

$$E(C_{m}) = -\phi_{10} \sum_{r=1}^{10} p_{mn} \ln(p_{mn})$$

$$\phi_{10} = \frac{1}{\ln(10)}$$

Where; $E(C_{m})$ is the measure of entropy by a discrete probability distribution for $C_{m}$, $\phi_{10}$ is the normalization factor that guarantees $0 \leq E(p_{1}, p_{2}, \ldots, p_{L}) \leq 1$, $p_{mn}$ is the probability distribution of $C_{m}$ for the diverse technical measures. Higher entropy or $(p_{1}, p_{2}, \ldots, p_{L})$ implies smaller variances and lesser information in a probability distribution $p_{L}$. At the end, bridge technical measure weights on each of the nine $C_{m}$ is calculated based on Chan and Wu (2005) equation (20):

$$e_{i} = \frac{E(C_{m})}{\sum_{m=1}^{q} E(C_{m})}$$

Where; $e_{i}$ is the importance weight of technical measure, and $E(C_{m})$ is the measure of entropy by a discrete probability distribution for $C_{m}$.

### 2.2 Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS)

Upon determination of technical measure weights, a multi-criteria decision making approach, TOPSIS, is undertaken. This approach takes into account the following criteria: (i) qualitative benefit; (ii) quantitative benefit; and (iii) cost criteria. As part of TOPSIS analysis, the following two most contradicting alternatives are surmised: (a) ideal alternative in which the maximum gain from each of the criteria values is taken; and (b) negative ideal alternative in which the maximum loss from each of the criteria values is taken. Towards the end, TOPSIS opts in
for the alternative that converges to the ideal solution and opts out from the negative ideal alternative. Prior to undertaking the multi-criteria decision making approach, a TOPSIS matrix is created based on equation (21):

$$X = (x_{ij})$$

(21)

Where; $X$ is the bridge users comparison matrix; and $x_{ij}$ is an “m x n” matrix; where, ‘m’ represents the technical measures and ‘n’ represents the bridge components that display the score of bridge user ‘i’ on bridge component ‘j’. TOPSIS analysis comprises the following consecutive five steps: (i) normalized decision matrix; (ii) weighted normalized decision matrix; (iii) ideal and negative ideal solutions; (iv) bridge components separation measures; and (v) relative closeness to ideal solution as shown in Fig. 5.

**FIG. 5: TOPSIS Process Flow**

In this study, Hwang et al. (1993) numerical methodology is deployed based on its direct applicability to ranking bridge MR&R priorities and proven reliability. Generating the normalized decision matrix is intended to convert various parametric dimensions into non-dimensional parameters to allow for contrasting among criteria using Hwang et al. (1993) equation (22):

$$r_{ij} = \frac{x_{ij}}{\sqrt{\sum x^2_{ij}}}$$

(22)

Where; $r_{ij}$ is the normalized scoring value of bridge users on bridge components. Afterwards, the development of a weighted decision matrix is obtained by multiplying the importance weights determined from equations (8) and (20) by its corresponding column of the normalized decision matrix obtained from equation (22) through the deployment of Hwang et al. (1993) equation (23):

$$v_{ij} = w_i \times r_{ij} \quad \text{where; } w_i = e_m \times e_i$$

(23)

Where; $v_{ij}$ is the weighted normalized element of the TOPSIS matrix, and $w_i$ is the final importance weight, $e_i$ is the importance weight of technical measure, and $e_m$ is the importance weight of bridge component. Afterwards, the ideal and negative ideal solutions are determined using Hwang et al. (1993) equations (24) and (25):
\[ A^* = \{ v^*_1, ..., v^*_{jv} \} \]  
\[ A = \{ v'_1, ..., v'_{jv} \} \]  

Where; \( A^* \) is the positive ideal solution; where \( v^*_j = \{ \max v_{ij} \} \) if \( j \in J \); minimum \( v_{ij} \) if \( j \in J^c \); \( A \) is the negative ideal solution where \( v'_j = \{ \min v_{ij} \} \) if \( j \in J \); maximum \( v_{ij} \) if \( j \in J^c \); where \( J \) is the set of positive attributes or criteria; and \( J^c \) is the set of negative attributes or criteria. Afterwards, bridge competitors’ separation measures from ideal and negative ideal solutions are calculated by using Hwang et al. (1993) equations (26) and (27):

\[ S^*_i = \left( \sum (v^*_j - v_{ij})^2 \right)^{1/2} \]  
\[ S'_i = \left( \sum (v'_j - v_{ij})^2 \right)^{1/2} \]

Where; \( S^*_i \) is the separation from the positive ideal solution; \( S'_i \) is the separation from the negative ideal solution; and \( i \) is the number of bridge competitors. Finally, relative closeness to ideal solution is calculated by using Hwang et al. (1993) equation (28):

\[ C^*_i = \frac{S'_i}{S^*_i + S'_i} \quad ; \quad 0 < C^*_i < 1 \]

Where; \( C^*_i \) is the relative closeness to positive ideal solution. The highest-ranked bridge component for MR&R priorities is the one with a corresponding \( C^*_i \) closest to the value of unity ‘1’.

### 2.3 Gamma Deterioration Model

Typically, bridge deteriorations are mainly caused by chemical and/or physical mechanisms that significantly affect infrastructure material characteristics and subsequent components. In this study, the deterioration of an aging bridge infrastructure is typically modelled as a function of its resistance capacity. The deterioration function is defined as per Noortwijk et al. (2007) in equation (29):

\[ D(t) = R_o - R(t_k) \]  

Where \( D(t) \) is the deterioration function, \( R_o \) is the initial resistance, and \( R(t_k) \) is the resistance at time \( t_k \). The deterioration function is assumed to be an ascending-order process with independent deterioration time intervals. For instance, suppose a sequence of shock load effects occur at discrete times such that the overall bridge service period is divided into independent time intervals. Hence, the resistance deterioration function, \( R(t_k) \), at time \( t_k \), is represented as equations (30) and (31) from Wang et al. (2015):

\[ R(t_k) = R_o \times D(t_k) \]  
\[ D(t_k) = 1 - \sum_{i=1}^{k} G_i \] where; \( k = 1, 2, ..., n \)

Where \( R(t_k) \) is the resistance deterioration function, \( R_o \) is the initial resistance; \( D(t_k) \) is the deterioration at time \( t_k \); and \( G_i \sim \text{Ga} (\gamma, \beta) \) denotes a gamma function with the shape parameter, \( \gamma \), and the scale parameter, \( \beta \). It is important to note that equation (30) is a descending-order process with a corresponding mean and variance calculated as per Wang et al. (2015) in equations (32-a), (32-b), and (32-c):
\[ \mu[D(t_k)] = 1 - \beta \times \sum_{i=1}^{k} \gamma_i \]  
(32-a)

\[ \sigma^2[D(t_k)] = \beta^2 \times \sum_{i=1}^{k} \gamma_i \]  
where; \( k = 1, 2, \ldots, n \)  
(32-b)

\[ \gamma^*_i = \kappa \times (t_i^\gamma - t_{i-1}^\gamma) \]  
(32-c)

Where \( \mu \) is the mean; \( D(t_k) \) is the deterioration at time \( t_k \); \( \sigma^2 \) is the variance, \( \gamma^*_i \) is the deterioration parameter; \( \beta \) is the scale parameter; and \( \kappa \) is the rate of deterioration. It is important to note that the scale and shape parameters presented herein are assigned as deterioration parameters of random variables and are determined independently.

### 2.4 Determination of Deterioration Function

Typically, bridge element conditions are evaluated by conducting site inspections based on municipal and/or national standards. These inspections contribute significantly towards the resistance deterioration condition of bridge elements and reflect their existing state which may be predicted as a ratio of the existing deterioration resistance to its initial resistance as per Wang et al. (2015), in equation (33):

\[ D(t_k) = \frac{R_k}{R_0} \]  
(33)

Where \( D(t_k) \) is the deterioration function at time \( t_k \); \( R_k \) is the current resistance deterioration function at time \( t_k \); and \( R_0 \) is the initial resistance. The existing resistance deterioration function \( R_k \), and the initial resistance \( R_0 \), are typically estimated according to bridge design manuals and national code standards. Bridge deterioration resistance is rarely assessed due to the high costs incurred, which implies that very little or no information on existing bridge resistance is available. Hence, this study proposes a numerical method to estimate deterioration parameters based on previous data of similar bridges.

#### 2.4.1 Estimation of Deterioration Parameters

In order to estimate the deterioration parameters \( (\gamma) \) and \( (\beta) \), the shape and scale deterioration function \( D(t_k) \) presented in equation (33) will be utilized to determine the deterioration of similar existing bridges \( k \), with a corresponding service life of \( t_1, t_2, \ldots, t_k \). By substitution, the deterioration function is presented as per Wang et al. (2015) in equation (34):

\[ 1 - D(t_i) = \beta \times \kappa \times (t_i^\gamma) \]  
where; \( i = 1, 2, \ldots, k \)  
(34)

Where \( D(t_i) \) is the deterioration at time \( t_i \); \( \gamma \) and \( \beta \) are the random shape and scale and deterioration parameters, and \( \kappa \) is the rate of deterioration. By taking the logarithmic for both sides of equation (34), the deterioration function is expressed as per Wang et al. (2015) in equation (35):

\[ \ln(1 - D(t_i)) = \ln(\beta \times \kappa) + \gamma \ln(t_i) \]  
(35)

Now, the deterioration parameters \( \gamma \) and \( \beta \) can be estimated graphically by utilizing a regression analysis of previous similar bridges’ deterioration data; where the slope \( \gamma \) is the ratio of \( \ln(1 - D(t_i)) \) to \( \ln(t_i) \) and the y-intercept is \( \beta \times \kappa \). In equation (32-b), the variance does not account for the dynamic nature of the temporal deterioration function. Hence, an average variance formulation is presented as per Wang et al. (2015) in equation (36):

\[ \hat{\beta}^2 \times \kappa \times \sum_{i=1}^{k} t_i^\gamma = \sum_{i=1}^{k} (D(t_i) - \hat{D}(t_i))^2 \]  
where ;  
(36)
\[
\hat{\beta} = \frac{\sum_{i=1}^{k} (D(t_i) - \hat{D}(t_i))^2}{\hat{\beta} \times \hat{k} \times \sum_{i=1}^{k} t_i^\gamma} \quad \text{and} \quad \hat{k} = \frac{\hat{\beta} \times \hat{k}}{\hat{\beta}}
\]

Where \( \hat{\gamma} \) and \( \hat{\beta} \) are the estimated shape and scale deterioration parameters respectively; \( \hat{k} \) is the estimated rate of deterioration; \( D(t_i) \) is the deterioration at time \( t_i \); and \( \hat{D}(t_i) \) is the estimated deterioration at time \( t_i \).

3. SYSTEM IMPLEMENTATION AND VALIDATION

The implementation of the decision support system is undertaken in two main steps; i) perturbation; and ii) quality evaluation. The algorithm is implemented as a probabilistic distribution function such that a random deterioration variable, \( D \), possesses a standard Gamma distribution of a distinguished shape parameter, \( \gamma \), and scale parameter, \( \beta \), defined as per Johnson et al. (1995) in equation (37):

\[
f_D(x) = \left( \frac{x}{\beta} \right)^{\gamma-1} \frac{1}{\beta \Gamma(\gamma)} e^{-\frac{x}{\beta}} \quad \text{where; } x, \gamma, \text{ and } \beta \geq 0
\]  

(37)

Where \( x \) is the deterioration parameter, \( \gamma \) is the shape parameter, \( \beta \) is the scale parameter, and \( \Gamma \) is the gamma function defined as per Johnson et al. (1995) in equation (38):

\[
\Gamma(\gamma) = \int_0^\infty x^{\gamma-1} e^{-x} dx
\]  

(38)

In this study, a gamma model with shape and scale parameters greater than zero is assumed to be a continuous stochastic model if the following conditions are satisfied: i) probability of \( D(0) = 0 \) is unity; ii) \( D(t) \) comprises independent deterioration increments; and iii) increments follow a gamma function such that the mean and variance are determined as per Johnson et al. (1995) in equation (39):

\[
\mu[D(t)] = \gamma \times \beta \quad \text{and} \quad \sigma^2[D(t)] = \gamma \times \beta^2
\]  

(39)

Where \( \mu \) is the mean, \( \sigma^2 \) is the variance, \( \gamma \) is the shape parameter, and \( \beta \) is the scale parameter.

3.1 Quality of Fit

Although regression analysis is capable of modeling a data scatter, significant variance may be noticed in the manner it represents the actual data value. Testing the quality of fit of a regression analysis trend line is typically conducted by either of the two following procedures: 1) heuristic, where manual inspection is conducted in parallel with an error minimization procedure; or 2) non-heuristic procedure, where hypothetical procedures such as the Chi-square test are deployed. In order to ease the use of regression analysis, the manual inspection of trend line fitting with an error minimization procedure is adopted since such fittings are automatically generated with advanced modeling software available in the market. The procedure is based on adjusting the fitted trend line to minimize the error. The sum (E) of the squares of differences between the actual and proposed trend line fit is then minimized to obtain the magnitude of adjustment factor that results in the best fit with the actual data scatter. The error minimization procedure is identified as per equation (40):

\[
E_{\text{min}} = \sum_{i=1}^{n} \left( \frac{d_{\text{act},i} - a(d_{\text{pro},i})}{d_{\text{act},i}} \right)^2
\]  

(40)

Where \( E_{\text{min}} \) is the minimized error, \( i = 1 \ldots n \) is the number of actual data scatters, \( d_{\text{act},i} \) is the actual data value at the \( i^{th} \) location, \( d_{\text{pro},i} \) is the proposed data value at the \( i^{th} \) location, and \( a \) is a scaling factor to be applied to
the proposed trend line. It is noted that the bracketed terms in equation (20) have been normalized with respect to
the average of actual data, \( \overrightarrow{d}_{act,i} \) as per equation (41):

\[
\overrightarrow{d}_{act,i} = \frac{1}{n} \sum_{i=1}^{n} d_{act,i}
\]  

Towards the end, it is important to note that the proposed trend line fit contributes towards an accurate estimation
of the shape and scale deterioration parameters such that error tolerances are respected.

3.1.1 Probabilistic Matrix Factorization

As part of enhancing dataset quality, collaborative filtering algorithms to determine interrelationships amongst
deterioration parameters are investigated. The matrix factorization approach is found to be the most effective
amongst the examined techniques due to its latent feature in determining the underlying correlations amongst
independent variables. In this study, a probabilistic matrix factorization technique is deployed to predict
deterioration datasets of existing bridges while overcoming biased and over-fitted values. The model-based
approach is undertaken by the following four main processes: (1) singular value matrix decomposition (SVMD);
(2) data normalization; (3) factorization; and (4) regularization. Firstly, the matrix decomposition process is
deployed to predict resistance deterioration values, \( \mu[ g(t)] \), of a bridge component as per Takács et al. (2008),
in equation (42):

\[
\hat{r}_{ij} = p_{ik}^T q_j = \sum_{k=1}^{k} p_{ik} q_{kj}
\]  

Where \( \hat{r}_{ij} \) is the predicted resistance deterioration; \( p_{ij}^T \) is the bridge preference factor vector; \( q_j \) is the resistance
deterioration factor vector; \( p_{ik} \) is the bridge preference factor matrix; and \( q_{kj} \) is the resistance deterioration factor
matrix such that the dot product of \( p_{ik} \) and \( q_{kj} \) approximates the \( \hat{r}_{ij} \). Afterwards, a gradient descent technique
is deployed in order to determine the bridge preference and resistance deterioration factor vectors \( p_{ij}^T \) and \( q_j \)
respectively. The error between the predicted and actual resistance deterioration value to obtain a local minima of
each ‘bridge-resistance deterioration’ pair is determined as per Takács et al. (2008) as in equation (43):

\[
e^2_{ij} = (r_{ij} - \hat{r}_{ij})^2 = (r_{ij} - \sum_k p_{ik} q_{kj})^2
\]  

Where; \( e^2_{ij} \) is the squared error difference; \( r_{ij} \) is the actual resistance deterioration; \( \hat{r}_{ij} \) is the predicted resistance
deterioration; \( p_{ik} \) is the bridge preference factor matrix; and \( q_{kj} \) is the resistance deterioration factor matrix. It
is important to note that the squared error of the predicted and actual resistance deterioration data is implemented
in order to account for over- or under-estimated values.

3.1.2 Error Minimization

In order to minimize the error value, a modification to \( p_{ik} \) and \( q_{kj} \) matrices is required to determine the value of
the gradient at its present state. Hence, a differentiation of equation (43) with respect to \( p_{ik} \) is deployed as per
Takács et al. (2008) in equation (44):

\[
\frac{\partial}{\partial p_{ik}} e^2_{ij} = -2(r_{ij} - \hat{r}_{ij})(q_{kj}) = -2e_{ij} q_{kj}
\]

\[
\frac{\partial}{\partial q_{kj}} e^2_{ij} = -2(r_{ij} - \hat{r}_{ij})(p_{ik}) = -2e_{ij} p_{ik}
\]  

Where \( e^2_{ij} \) is the squared error difference; \( e_{ij} \) is the error difference; \( r_{ij} \) is the actual resistance deterioration; \( \hat{r}_{ij} \)
is the predicted resistance deterioration; \( p_{ik} \) is the bridge preference factor matrix; and \( q_{kj} \) is the resistance
deterioration factor matrix. Upon determination of the gradient descent value, the differentiation of equation (23) is rearranged as per Takács et al. (2008) in equation (45):

\[
p'_{ik} = p_{ik} + \alpha \frac{\partial}{\partial p_k} e^2_{ij} = p_{ik} + 2\alpha e_{ij} q_{kj}
\]

\[
q'_{kj} = q_{kj} + \alpha \frac{\partial}{\partial q_{kj}} e^2_{ij} = q_{kj} + 2\alpha e_{ij} p_{ik}
\]

Where \( p'_{ik} \) is the differentiated bridge preference factor matrix; \( q'_{kj} \) is the differentiated resistance deterioration factor matrix; \( e^2_{ij} \) is the squared error difference; \( \alpha \) is the gradient descent rate factor; \( e_{ij} \) is the error difference; \( p_{ik} \) is the bridge preference factor matrix; and \( q_{kj} \) is the resistance deterioration factor matrix. It is important to note that the \( \alpha \) factor in equation (45) is the tolerance value that defines the rate of gradient descent approaching the minimum. In order to avoid excessive oscillations and bypassing the local minima, a modification factor \( \alpha \), with a value of 0.0002 is assumed. In this study, the error minimization procedure is proposed for the bridge-deterioration pairs. For instance, let \( N \) be a finite ordered set of training data in the form of \( ( q_{kj}, p_{ik}, \hat{r}_j ) \), the error, \( e_{ij} \), for each iterative dataset will be minimized when the connotations amongst the attributes is learnt. Afterwards, the error minimization process is concluded when the iteratively determined error converges to its minimum as per Takács et al. (2008) in equation (46):

\[
E = \sum_{(q_{kj}, p_{ik}, \hat{r}_j) \in N} (r_{ij} - \sum_k p_{ik} q_{kj})^2
\]

Where \( E \) is the minimized error value; \( q_{kj} \) is the resistance deterioration factor; \( p_{ik} \) is the bridge preference factor matrix; \( \hat{r}_j \) is the predicted resistance deterioration; and \( r_{ij} \) is the actual resistance deterioration.

3.1.3 Regularization

In order to avoid dataset over-fitting, a regularization process is implemented by incorporating a parameter factor \( \gamma \), to regularize the magnitudes of the bridge-deterioration resistance factor vectors. Also, a regularization parameter \( \gamma \) with a value of 0.02 is assumed in order to avoid large number approximations and achieve a better approximation of the bridge deterioration resistance capacity. The squared-error difference between the predicted and actual resistance deterioration value to obtain a local minima of each ‘bridge-resistance deterioration’ pair is rearranged as per Takács et al. (2008) in equation (47):

\[
e^2_{ij} = (r_{ij} - \hat{r}_j)^2 = (r_{ij} - \sum_k p_{ik} q_{kj})^2 + \frac{\gamma}{2} \sum_k (\|p_{ik}\|^2 + \|q_{kj}\|^2)
\]

Where \( e^2_{ij} \) is the squared error difference; \( r_{ij} \) is the actual resistance deterioration; and \( \hat{r}_j \) is the predicted resistance deterioration; \( p_{ik} \) is the bridge preference factor matrix; and \( q_{kj} \) is the resistance deterioration factor matrix. Upon determination of the squared error difference, the differentiation of the equation (43) is rearranged as per Takács et al. (2008) in equation (48):

\[
p'_{ik} = p_{ik} + \alpha \frac{\partial}{\partial p_k} e^2_{ij} = p_{ik} + \alpha(2e_{ij} q_{kj} - \gamma p_{ik})
\]

\[
q'_{kj} = q_{kj} + \alpha \frac{\partial}{\partial q_{kj}} e^2_{ij} = q_{kj} + \alpha(2e_{ij} p_{ik} - \gamma q_{kj})
\]

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Where $p'_{ik}$ is the differentiated bridge preference factor matrix; $q'_{ij}$ is the differentiated resistance deterioration factor matrix; $e^2_{ij}$ is the squared error difference; $\alpha$ is the gradient descent rate factor; $\gamma$ is the regularization parameter; $e_{ij}$ is the error difference; $p_{ik}$ is the bridge preference factor matrix; and $q_{ij}$ is the resistance deterioration factor matrix.

3.2 System Validation

To validate the workability of the proposed system, a case study of a bridge in Ottawa, Canada composed of a concrete box-girder with a total span of 200 ft. supported with a central interior bent at 100 ft. is developed in CSiBridge as illustrated in Fig. 6. The challenge underlying the system validation is to provide priority ranking to MR&R decisions for the diverse bridge components.

**FIG. 6: Conceptual Bridge Design Information Model**

Prior to inputting project related data into BrIM tool, the following list summarizes main parametric design assumptions: 1) Abutment: skewed at 15 degrees and supported at bottom girder only; 2) Pre-stressing: 4 nos. 5 in\(^2\) tendons with a 1,080 kips capacity each; 3) Interior bent: 3 nos. 5 ft square columns; 4) Deck: parabolic variation ranging from 5-10 ft in nominal depth; 5) Pile cap: 3 nos. 13’ x 13’ x 4’; and 6) Pile: 9 nos. 14” dia. steel pipe filled with concrete reinforced with 8 nos. of #9 reinforcement bars at each pile cap. It is important to note that the aforementioned assumptions are made based on normal job conditions. However, if geographical constraints are encountered, these factors may increase or decrease accordingly. For example, if the job terrain encountered is rough, substructure concrete and pile design factors will increase and subsequently significantly influence overall project cost. The systematic procedure of the integrated system is demonstrated in a step-by-step process in Figs. 7 through 12 which present snapshots of the proposed system modules. Fig. 6 presents the integrated deterioration gateway module.
FIG. 7: Deterioration System Gateway Module

Once the user selects the desired deterioration module, the system displays a module within the activity main module where the user inputs importance rating on bridge components as shown in Fig. 8.

FIG. 8: Bridge User Importance Rating on Bridge Components

Following bridge user’s assessment on the importance of relative bridge components on bridge alternatives, relative importance perception ratings in crisp and fuzzy forms are obtained according to equations (1) and (2). Fig. 9 illustrates the rating for the Stakeholders/Government; whereas similar rating is conducted for the other users.
A sample of the developed algorithm used to extract the Stakeholder/Government assessment and relative importance ratings is shown below:

```csharp
private void frmQualitFunctionDeployment_Load(object sender, EventArgs e)
{
    //this.macTrackBar1.Height = 91;
    tblStakeholders_Measure = FC.Con.GetDataTable("Select * from OfdReportBeneficiaries order by [Whats]"); tblStakeholders_Measure.TableName = "tblStakeholders_Measure"
    tblStakeholders_Number = tblStakeholders_Measure.Copy();
    tblStakeholders_Number.TableName = "tblStakeholders_Number"
    tblStakeholders_Fuzzy = tblStakeholders_Measure.Copy();
    tblStakeholders_Fuzzy.TableName = "tblStakeholders_Fuzzy"
    dataGridView1.DataSource = tblStakeholders_Measure;
    public void RelativeImportance(ref DataTable tbl1, ref DataTable tbl1_1, DataTable tblNumber, DataTable tblFuzzy)
    {
        List<decimal> RelImp = new List<decimal>();
        List<decimal> RelImp1 = new List<decimal>();
        for (int i = 0; i < tblNumber.Rows.Count; i++)
        {
            DataRow dr = tbl1.NewRow();
            dr[0] = tblNumber.Rows[i][0];
            tbl1.Rows.Add(dr);
        }
    }
}
```

Afterwards, a bridge users’ competitive matrix is developed based on equations (3) and (4). Then, the probability distribution and corresponding measure of entropy of bridge components is determined by using equations (5) through (8) as shown in Fig. 10.

FIG. 9: Relative Importance Rating on Bridge Components
FIG. 10: Bridge Users Competitive Matrix on Bridge Components

Once completed, the user can proceed with inputting a set of improvement goals for bridge components as illustrated in Fig. 11.

FIG. 11: Improvement Goals for Bridge Components

Following the input of goals, the user can proceed with TOPSIS operations by clicking on TOPSIS matrix to develop priority rankings of bridge components as shown in Figs. 12 and 13.

FIG. 12: TOPSIS Analysis Module
As shown in Fig. 13, bridge components ‘C1. Approach Slab; ‘C5. Girder’; and ‘C6. Parapet’ possess maximum weights followed by ‘C8. Pier; ‘C7. Expansion Joint; ‘C9. Foundation; and ‘C2. Deck Slab; while, ‘C6. Bearings and ‘C7. Abutment’ components possess the minimum weights. Typically, bridges are designed while taking into account the following main criteria: (1) Girder; (2) Pier; and (3) Foundation. However, by deploying the complex quality function technique, it is determined that incorporating additional bridge users, such as contractors/builders and public/residents, influence bridge components importance weights; and hence, explicitly implying a more realistic and practical decision support system. With approach slab and girder components being the most expensive and contribute significantly towards construction costs, it has been determined that its importance weight is at the highest rank. On the other hand, the bearings and abutment components are determined to possess the least importance weight. Typically, bridge piers and foundation have been ranked first at the bridge conceptual design stage since they are the major components for bridge projects. However, in this study, bridge pier and foundation are ranked second based on bridge users’ relative importance perception scorings. This implies that bridge users did not anticipate deterioration on piers to affect bridge performance as opposed to the approach slab, girder, and parapet components. A sample of the developed algorithm used to determine bridge components TOPSIS rankings is shown below.

```csharp
for (int i = 0; i < tblMatrix.Rows.Count; i++)
{
    decimal Sum_P11 = 0;
    for (int Cols = 1; Cols < tblMatrix.Columns.Count; Cols++)
    {
        if (Cols < 10)
        {
            if (Val(tblMatrix.Rows[i][Cols].ToString()) != 0)
            {
                decimal p_11 = Math.Round(Val(tblMatrix.Rows[i][Cols].ToString()) / Val(tblMatrix.Rows[i][10].ToString()), 4);
                p_11 = Math.Round(p_11 * (decimal)Math.Log((double)p_11), 4);
                Sum_P11 += p_11;
            }
        }
        else
    }
}
```
decimal sum = tblMatrix.AsEnumerable().Sum(x => x.Field<decimal>(10));

foreach (DataRow dr in tblMatrix.Rows)
{
    if (Val(dr[10].ToString()) != 0)
        dr[10] = Math.Round(Val(dr[10].ToString()) / sum, 2);  //--e1
}

return tblMatrix;

Following the determination of components rankings, the user is guided to the HOWs scoring input form where the user inputs importance rating on bridge maintenance, repair, and replacement alternatives as shown in Fig. 14.

**FIG. 14:** Bridge User Importance Rating on MR&R Alternatives

Following bridge user’s assessment on the importance of relative bridge components on bridge MR&R alternatives, relative importance perception ratings in crisp and fuzzy forms are obtained according to equations (1) and (2). Afterwards, a bridge users’ competitive matrix is developed based on equations (3) and (4). Then, the probability distribution and corresponding measure of entropy along with competitive ranking of bridge components is determined by using equations (5) through (8) and (11) respectively as shown in Fig. 15.

**FIG. 15:** Bridge Components Competitive Matrix on MR&R Alternatives

It is important to note that improvement goals to evaluate the competitiveness rating amongst bridge components for diverse bridge type alternatives are set for each component. Accordingly, corresponding improvement ratios and competitive ratings are determined. In comparison with Fig. 10, bridge component ‘C_1’ possesses the second highest importance weight and competitive rating as opposed to ‘C_7’ which possess the first ranking from an importance standpoint; however, ‘C_5’ had dropped to the seventh ranking in terms of competitiveness. A similar analogy is observed for the other bridge components; such as ‘C_3’ and ‘C_4’. Once completed, a TOPSIS matrix is
developed based on equation (21). Afterwards, normalized decision and weighted matrices are constructed as per equations (22) and (23) respectively. Next, the determination of positive and negative ideal solutions is undertaken as per equations (24) and (25) respectively and set as the reference datum. Towards the end, separations from positive and negative ideal solutions are obtained as per equations (26) and (27) respectively. Finally, TOPSIS relative closeness to ideal solution decision matrix is obtained according to equation (28) with priority ratings as illustrated in Fig. 16.

![TOPSIS Matrix](image)

**FIG. 16: Final Ranking of Bridge MR&R Alternatives**

As illustrated in Fig. 16, ‘C1. Approach Slab’ is the bridge component that requires further consideration at the conceptual design stage. Besides that, the MR&R solution ‘S1. M15’, which implies the maintenance solution when the approach slab component extent of deterioration is at 15%, is the most favorable. On the other hand, ‘REPA60’, which implies the repair solution when approach slab component extent of deterioration is at 60%, is the MR&R solution with the second rank. Based on the aforementioned, it is shown that bridge users have no preference to maintenance works when the approach slab extent of deterioration exceeds 15%. Furthermore, the deterioration resistance capacity of the approach slab must be reconsidered at the conceptual design stage in order to withstand a 60% extent of deterioration. Once completed, a deterioration model for each of the bridge components is necessary to predict their time-dependent deterioration behavior. Hence, mean values of the resistance function for the bridge ‘approach slab’ is obtained from previous similar bridges at diverse years throughout their service life. Table 2 summarizes approach slab mean deterioration resistance data at diverse years.

**TABLE 2: Approach Slab Mean Deterioration Resistance Data at Diverse Years**

<table>
<thead>
<tr>
<th>Time, ( t ) (years)</th>
<th>Bridge 1* ( \mu[g(t)] ) (%)</th>
<th>Bridge 2* ( \mu[g(t)] ) (%)</th>
<th>Bridge 3* ( \mu[g(t)] ) (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>11</td>
<td>-</td>
<td>20.3</td>
<td>11.0</td>
</tr>
<tr>
<td>12</td>
<td>26.2</td>
<td>41.2</td>
<td>12.8</td>
</tr>
<tr>
<td>13</td>
<td>21.4</td>
<td>-</td>
<td>10.3</td>
</tr>
<tr>
<td>14</td>
<td>24.3</td>
<td>24.4</td>
<td>-</td>
</tr>
<tr>
<td>15</td>
<td>-</td>
<td>38.3</td>
<td>22.4</td>
</tr>
<tr>
<td>16</td>
<td>25.7</td>
<td>23.8</td>
<td>13.4</td>
</tr>
<tr>
<td>17</td>
<td>-</td>
<td>26.8</td>
<td>16.4</td>
</tr>
<tr>
<td>18</td>
<td>10.4</td>
<td>27.6</td>
<td>15.8</td>
</tr>
<tr>
<td>19</td>
<td>26.2</td>
<td>-</td>
<td>15.5</td>
</tr>
<tr>
<td>20</td>
<td>-</td>
<td>33.8</td>
<td>-</td>
</tr>
<tr>
<td>21</td>
<td>-</td>
<td>34.4</td>
<td>19.1</td>
</tr>
<tr>
<td>22</td>
<td>17.4</td>
<td>48.3</td>
<td>-</td>
</tr>
<tr>
<td>23</td>
<td>-</td>
<td>34.5</td>
<td>19.8</td>
</tr>
<tr>
<td>24</td>
<td>-</td>
<td>39.2</td>
<td>22.3</td>
</tr>
<tr>
<td>25</td>
<td>44.1</td>
<td>50.3</td>
<td>29.4</td>
</tr>
</tbody>
</table>

*Source: Ministry of Transportation, Highway and Bridges, Ontario*
Upon obtaining mean resistance data, the user can then proceed with mean resistance deterioration module as shown in Fig. 17.

![Mean Resistance Deterioration Module](image)

**FIG. 17: Mean Resistance Deterioration Module**

Afterwards, the user inputs the year and corresponding mean deterioration percentages such that a regression analysis along with the quality of fit methodology is deployed as illustrated in Fig. 18.

![Approach Slab Regression Analysis at 15% Deterioration](image)

**FIG. 18: ‘Approach Slab’ Regression Analysis at 15% Deterioration**

As shown in Fig. 18, a probabilistic matrix factorization process is deployed to avoid biased and over-fitted values. The proposed technique predicts missing data from Table 2. The predicted data is plotted in a scatter fit where a regression analysis is conducted in order to determine the best of fit. Based on equations (38) and (39), the deterioration parameters, \( \hat{\gamma} \) and \( \hat{\beta} \), are estimated to be 1.1944 and \( \hat{\beta} \times \hat{K} = e^{-4.7878} = 0.00833 \) respectively; where \( \hat{\beta} \) and \( \hat{K} \) are determined as per equation (40) and equal to 0.0085 and 0.978 respectively as per the minimized error-fitted trend line. Once completed, the proposed system presents a recommendation statement to
reconsider the performance of the approach slab at the conceptual design stage in order to enhance its corresponding deterioration resistance capacity at the age of 9 years as shown in Fig. 18. A sample of the developed algorithm used to determine bridge component deterioration resistance capacity and regression analysis is shown below.

```csharp
for (int x = 0; x < dataGridView1.RowCount; x++)
{
    for (int y = 0; y < dataGridView1.ColumnCount - 1; y++)
    {
        test[x, y] = double.Parse(dataGridView1.Rows[x].Cells[y + 1].Value.ToString());
    }
}
NonnegativeMatrixFactorization Test = new NonnegativeMatrixFactorization(test, dataGridView1.ColumnCount - 1);
double[,] actual = Matrix.Multiply(Test.LeftNonnegativeFactors, Test.RightNonnegativeFactors);
for (int x = 0; x < dataGridView1.RowCount; x++)
{
    for (int y = 0; y < dataGridView1.ColumnCount - 1; y++)
    {
        if (actual[y, x] > 1)
        {
            Random random = new Random();
            var next = random.NextDouble();
            actual[y, x] = (next * (0.999));
        }
        dataGridView1.Rows[x].Cells[y + 1].Value = Math.Round(actual[y, x], 2);
    }
}
string AnticipatedCost = "0.00";
lbBetaValue.Text = "Beta = " + Math.Round(decimal.Parse(Result[0, 3].ToString()), 4);
LbKValue.Text = "K = " + Math.Round(decimal.Parse(Result[0, 4].ToString()), 4);
LbGammaValue.Text = "Gamma = " + Math.Round(decimal.Parse(Result[0, 5].ToString()), 4);
// LbTimeValue1.Text = "Time of Interference = " + Result[0, 6].ToString();
LbTimeValue2.Text = Math.Round(decimal.Parse(Result[0, 7].ToString()), 2).ToString();
if (Properties.Settings.Default.ReconsiderValue.ToLower() == "approach slab")
{
}
```
4. DISCUSSION AND CONCLUSION
The proposed system presented herein is capable of forecasting bridge components elemental degradation at the conceptual design stage where very little or no available information about a bridge project is available. However, the results presented herein are intended for life cycle cost analysis and may be incorporated for allocating preventative maintenance budget at the conceptual design stage of a bridge project with an anticipated accuracy level ranging between a minimum of -15% to a maximum of +15% approximately. Furthermore, future studies may utilize the findings of this study to enhance the performance of bridge components. In this study, the case study presented herein is used to validate the accuracy of the proposed system; where the results obtained are verified with findings of similar bridges and shared with experienced qualified asset managers and found to be of acceptable form. One limitation of the proposed system; however, underlies the shortage of similar bridges deterioration resistance data which could significantly affect the predicted regression fit and corresponding life cycle cost analysis. Towards the end, the forecasted bridge deteriorations are compared to the actual bridge deteriorations and found to be within a percentage difference ranging from approximately 10 to 15%, as illustrated in Table 3.

<table>
<thead>
<tr>
<th>Year</th>
<th>System Data (%)</th>
<th>Actual Data* (%)</th>
<th>Percentage Difference (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>4.5</td>
<td>5.2</td>
<td>13.5</td>
</tr>
<tr>
<td>6</td>
<td>9.4</td>
<td>10.7</td>
<td>12.1</td>
</tr>
<tr>
<td>7</td>
<td>12.6</td>
<td>14.7</td>
<td>14.3</td>
</tr>
<tr>
<td>8</td>
<td>13.7</td>
<td>16.2</td>
<td>15.4</td>
</tr>
<tr>
<td>9</td>
<td>15.3</td>
<td>17.1</td>
<td>10.5</td>
</tr>
</tbody>
</table>

*Source: Ministry of Transportation, Highway and Bridges, Ontario

Prior to comparison of results, it is important to note that the discrepancy between the results is due to the multiple assumptions made as well as to the availability of deterioration data for similar bridge projects. For instance, deterioration forecast is based on moderate weather conditions. If severe conditions occur, the forecasting result would be instantly affected. Also, the deterioration forecast is estimated based on a probabilistic matrix factorization approach. The range of variation in maintenance deterioration results is somehow acceptable at the conceptual design stage since the overall information pertaining to the project is neither fully defined nor detailed. Overall results show that the accuracy of the system varies depending on:
1. Subjectivity,
2. Methodology,
3. Project definition,
4. Weather conditions, and
5. Availability of historical data for similar bridges.

In this study, an integrated fuzzy logic decision support system with bridge information management system (BrIMS) in order to assist bridge stakeholders and engineers/designers predict bridge MR&R decisions is proposed. Comparative analyses of diverse bridge components are conducted utilizing complex QFD and TOPSIS systematic approaches to assist users in predicting MR&R decisions at the conceptual design stage. The proposed system is then validated through a case study and is presently under further development in a .NET framework. When comparing practical results obtained to existing design guidelines, it is the found that the proposed system is capable of providing the user with recommendation statements to enhance the performance of bridge component at a particular year throughout its recovery period. Furthermore, the MR&R alternatives are defined as cost representatives of a bridge component. In this study, practical results of components’ deterioration forecasts fall within Class 3 of the AACE International Cost Estimate Classification System with an accuracy range of 10% to 30% which is an improvement to bridge components’ deterioration forecast at the conceptual design stage that typically fall within Class 4 with an accuracy range of 20% to 50%. This indicates that the system may be utilized...
to enhance a bridge component deterioration resistance capacity for better performance in terms of maintenance, repair, and replacement costs despite achieving the required integrity and soundness of a bridge component in accordance with existing design guidelines.

Furthermore, the proposed system is found to possess an advantage over existing decision support algorithms and deterioration forecast applications known to the industry by including the following distinguishing features:

- Developing an integrated stand-alone all-in-one system capable of providing the user with recommendations for bridge components MR&R alternatives based on a combination of decision support, probabilistic matrix factorization, and deterioration forecast at the conceptual design stage.
- Facilitating the interoperability and compatibility among the diverse modules, sub-modules and database resources.
- Applicability of the proposed system for the following list of bridge categories anywhere around the globe: (1) beam bridges; (2) truss bridges; (3) cantilever bridges; (4) arch bridges; (5) tied-arch bridges; (6) suspension bridges; (7) cable-stayed bridges; (8) movable bridges; and (9) double-decked bridges regardless of the differences noted in design codes among bridge asset management authorities as system databases presented in this study are designed in such a way that they may be customized to suit accordingly.
- Implementing the TOPSIS technique for BrIM model to assist in prioritizing preventative maintenance, repair, and replacement decisions along with the deployment of complex cyclic gamma shock models for prediction of bridge temporal deteriorations at the conceptual design stage.

Given the scarcity of studies on integrations of fuzzy logic decision support systems with bridge information management systems, the authors are conducting further studies in that field to include lifecycle cost analysis of bridge components. Furthermore, more attention is focused towards the effect of incorporating complex quality functions on prioritizing MR&R decisions for bridge components. The authors are presently working on the expansion of the probabilistic and numerical model databases of solutions, which is an important step towards developing rational design rules for bridge components.

Finally, it is concluded that the proposed system possesses design and prediction limitations pertaining to complex and combined bridge sub- and super-structure designs. It is necessary to mention that the proposed system is developed as a validation tool that may be utilized to predict and prioritize MR&R decisions for components for diverse bridge alternatives. The proposed system may be utilized in the design of bridge projects compiled with BrIMS integration. This capability provides the system a great advantage over other management information algorithms, prototypes, or models published in the literature. Also, the results presented in this study are anticipated to be of major significance to the bridge construction industry and would be a novel contribution to advancements in BrIMS integrations with deterioration forecast at the conceptual design stage of bridge projects.

5. REFERENCES


Lounis Z, and Madanat S. (2002). Integrating mechanistic and statistical deterioration models for effective bridge management. 7th International Conference on Applications of Advanced Technology in Transportation, ASCE, Boston, MA, 513-520.


